ApplMath18

Ninth Conference on Applied
Mathematics and Scientific Computing

17-20 September 2018, Solaris, Šibenik, Croatia

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Main information

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Ninoslav Truhar, *University of Osijek*

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Sponsors

- Croatian Academy of Sciences and Arts
- Faculty of Science, University of Zagreb
- Ministry of Science and Education of the Republic of Croatia

Plenary Speakers

Jose Antonio Carrillo de la Plata, *Imperial College, London*

Lars Grasedyck, *RWTH Aachen*

Daniel Kressner, *EPFL*

Andro Mikelić, *Université Lyon 1*

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Gianluigi Rozza, *SISSA - International School for Advanced Studies*

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68. Jerome Weston, *University of Dubrovnik*

69. Ana Žgaljić Keko, *Faculty of Electrical Engineering and Computing, University of Zagreb*

70. Bojan Žugec, *Faculty of Organization and Informatics, University of Zagreb*

Programme

Opening and closing, as well as all plenary lectures take place in the conference room **Krka 1**.

Monday, September 17th

09:00 - 09:15	Opening	
09:15 - 10:05	Plenary: Nečasová	
10:05 - 10:30	Coffee break	
	Krka 1	**Krka 2**
10:35 - 11:00	Marušić-Paloka	Jaklič
11:00 - 11:25	Radošević	T. Bosner
11:25 - 11:50	Sabbagh	M. Marušić
11:50 - 12:15	Caggio	Rodrigues
12:30 - 14:00	Lunch	
14:10 - 15:00	Plenary: Kressner	
	Krka 1	**Krka 2**
15:10 - 15:35	Begović Kovač	Kalousek
15:35 - 16:00	Drmač	Dražić
16:00 - 16:25	Coffee break	
	Krka 1	**Krka 2**
16:30 - 16:55	Žgaljić-Keko	Martins
16:55 - 17:20	Vrbaški	Kunštek
17:20 - 17:45	Rabar	Crnjac

Tuesday, September 18th

09:00 - 09:50	Plenary: Grasedyck	
09:50 - 10:40	Plenary: Süli	
10:40 - 11:05	Coffee break	
	Krka 1	**Krka 2**
11:10 - 11:35	Bujanović	Pažanin
11:35 - 12:00	Grubišić	Galić
12:00 - 12:25	N. Bosner	Radulović
12:30 - 14:00	Lunch	
14:10 - 15:00	Plenary: Carrillo	
	Krka 1	**Krka 2**
15:10 - 15:35	Bukal	Truhar
15:35 - 16:00	Kovač	Šain Glibić
16:00 - 16:25	Coffee break	
	Krka 1	**Krka 2**
16:30 - 16:55	Valdman	Vlah
16:55 - 17:20	Tavares	Ogata
17:20 - 17:45	Bužančić	Abdullah
17:45 - 18:10	da Cruz	Mohd Mahali

Wednesday, September 19th

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09:50 - 10:15	Coffee break	
	Krka 1	**Krka 2**
10:20 - 10:45	Veselić	Črnjarić-Žic
10:45 - 11:10	Nakić	Franck
11:10 - 11:35	Lazar	Maćešić
11:35 - 12:00	Silva	Crnković
12:00 - 13:30	Lunch	
13:40 - 14:30	Plenary: Rozza	
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14:35 - 15:00	Arjmand	Erceg
15:00 - 15:25	Karatzas	Vojnović
15:25 - 19:25	Free time / Excursion	
19:30 - 00:00	Conference dinner	

Thursday, September 20th

09:00 - 09:50	Plenary: Mikelić	
09:50 - 10:15	Coffee break	
	Krka 1	**Krka 2**
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10:45 - 11:10	Jankov	Miodragović
11:10 - 11:35	Ljulj	Mililoža Pandur
11:35 - 12:00	Žugec	Periša
12:00 - 12:20	Coffee break	
	Krka 1	**Krka 2**
12:20 - 12:45	Pašić	Puvača
12:45 - 13:10	Weston	Carević
13:10 - 13:20	Closing	
13:30 - 14:30	Lunch	

Invited Talks

Attractive-repulsive models in collective behavior and applications

Jose Antonio Carrillo

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We will discuss properties of solutions to aggregation-diffusion models appearing in many biological models such as cell adhesion, organogenesis and pattern formation. We will concentrate on typical behaviours encountered in systems of these equations assuming different interactions between species under a global volume constraint.

Computing in Low Rank Tensor Formats in Sublinear Complexity

Lars Grasedyck

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TBA

Fast algorithms from low-rank updates

Daniel Kressner
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The development of efficient numerical algorithms for solving large-scale linear systems is one of the success stories of numerical linear algebra that has had a tremendous impact on our ability to perform complex numerical simulations and large-scale statistical computations. Many of these developments are based on multilevel and domain decomposition techniques, which are intimately linked to Schur complements and low-rank updates of matrices. These tools do not carry over in a direct manner to other important linear algebra problems, including matrix functions and matrix equations. In this talk, we describe a new framework for performing low-rank updates of matrix functions. This allows to address a wide variety of matrix functions and matrix structures, including sparse matrices as well as matrices with hierarchical low rank and Toeplitz-like structures. The versality of this framework will be demonstrated with several applications and extensions. This talk is based on joint work with Bernhard Beckermann, Stefano Massei, Leonardo Robol, and Marcel Schweitzer.

A monolithic phase-field model of a fluid-driven fracture in a nonlinear poroelastic medium

C. J. van Dujin, Andro Mikelić[1], M. F. Wheeler, T. Wick
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In this talk we present a full phase field model for a fluid-driven fracture in a nonlinear poroelastic medium. The poroelastic medium contains an incompressible elastic skeleton and the pores a filled with an incompressible viscous fluid. The regime is quasi-static and the permeability depends on the porosity, are being itself a function of the skeleton volume strain. In the previous work by the same authors (Comp. Geosc. 2015) a fully coupled system where the pressure is determined simultaneously with the displacement and the phase field, was considered for the linearizedquasi-static Biot equations. For the new model, we establish existence of a solution to the incremental problem through convergence of a finite dimensional approximation. Furthermore, we construct the corresponding Lyapunov functional that is linked to the free energy. Computational results are provided that demonstrate the effectiveness of this approach in treating fluid-driven fracture propagation. Specifically, our numerical findings confirm differences with test cases using the linear Biot equations.

On the problem of the motion of a rigid body with a cavity filled with viscous compressible fluid

G. P. Galdi, V. Mácha, Šarka Nečasová[1]
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We study the motion of the system, $\mathcal{S}$, constituted by a rigid body, $\mathcal{B}$, containing in its interior a viscous compressible fluid, and moving in absence of external forces. Our main objective is to characterize the long time behavior of the coupled system body-fluid. Under suitable assumptions on the "mass distribution" of $\mathcal{S}$, and for sufficiently "small" Mach number and initial data, we show that every corresponding motion (in a suitable regularity class) must tend to a steady state where the fluid is at rest with respect to $\mathcal{B}$. Moreover, $\mathcal{S}$, as a whole, performs a uniform rotation around an axis parallel to the (constant) angular momentum of $\mathcal{S}$, and passing through its center of mass.

References

[1] G. P. Galdi, V. Mácha, and Š. Nečasová: On the Motion of a Body with a Cavity Filled with Compressible Fluid, Submitted

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Reduced Order Methods: state of the art and perspectives with a special focus on Computational Fluid Dynamics

Gianluigi Rozza
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In this talk, we provide the state of the art of Reduced Order Methods (ROM) for parametric Partial Differential Equations (PDEs), and we focus on some perspectives in their current trends and developments, with a special interest in parametric problems arising in offline-online Computational Fluid Dynamics (CFD). Systems modelled by PDEs are depending by several complex parameters in need of being reduced, even before the computational phase in a pre-processing step, in order to reduce parameter space. Efficient parametrizations (random inputs, geometry, physics) are very important to be able to properly address an offline-online decoupling of the computational procedures and to allow competitive computational performances. Current ROM developments in CFD include: a better use of stable high fidelity methods, considering also spectral element method, to enhance the quality of the reduced model too; more efficient sampling techniques to reduce the number of the basis functions, retained as snapshots, as well as the dimension of online systems; the improvements of the certification of accuracy based on residual based error bounds and of the stability factors, as well as the the guarantee of the stability

of the approximation with proper space enrichments. For nonlinear systems, also the investigation on bifurcations of parametric solutions are crucial and they may be obtained thanks to a reduced eigenvalue analysis of the linearised operator. All the previous aspects are very important in CFD problems to be able to focus in real time on complex parametric industrial and biomedical flow problems, or even in a control flow setting, and to couple viscous flows -velocity, pressure, as well as thermal field - with a structural field or a porous medium, thus requiring also an efficient reduced parametric treatment of interfaces between different physics. Model flow problems will focus on few benchmark cases in a time-dependent framework, as well as on simple fluid-structure interaction problems or flow control problems in environmental sciences or medicine. Further examples of applications will be delivered concerning shape optimisation applied to industrial problems.

Implicitly constituted fluid flow models: analysis and approximation

Endre Süli

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Classical models describing the motion of Newtonian fluids, such as water, rely on the assumption that the shear stress is a linear function of the symmetric part of the velocity gradient of the fluid. This assumption leads to the Navier–Stokes equations. It is known however that the framework of classical continuum mechanics, built upon an explicit constitutive equation for the shear stress, is too narrow to describe inelastic behavior of solid-like materials or viscoelastic properties of materials. Our starting point in this work is therefore a generalization of the classical framework of continuum mechanics, called the *implicit constitutive theory*, which was proposed recently in a series of papers by Rajagopal. The underlying principle of the implicit constitutive theory in the context of viscous flows is the following: instead of demanding that the shear stress is an explicit (and, in particular, linear) function of the symmetric part of the velocity gradient, one may allow a nonlinear, implicit and not necessarily continuous relationship between these quantities. The resulting general theory therefore admits non-Newtonian fluid flow models with implicit and possibly discontinuous power-law-like rheology.

We develop the analysis of finite element approximations of implicit power-law-like models for viscous incompressible fluids. The shear stress and the symmetric part of the velocity gradient in the class of models under consideration are related by a, possibly multi-valued, maximal monotone graph. Using a variety of weak compactness techniques we show that a subsequence of the sequence of finite element solutions converges to a weak solution of the problem as the discretization parameter, measuring the granularity of the finite element triangulation, tends to zero. A key new technical tool in our analysis is a finite element counterpart of the Acerbi–Fusco Lipschitz truncation of Sobolev functions.

The talk is based on a series of joint papers with Lars Diening (Bielefeld), Christian Kreuzer (Dortmund), and Tabea Tscherpel (Oxford).

Reaction-diffusion models: dynamics and control

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Reaction-diffusion equations are ubiquitous and its applications include combustion and population dynamics modelling.

There is an extensive mathematical literature addressing the analysis of steady state solutions, traveling waves, and their stability, among other properties.

Control problems arise in many applications involving these models. And, often times, they involve control and/or state constraints, as intrinsic requirements of the processes under consideration.

In this lecture we shall present the recent work of our team on the Fisher-KPP and Allen-Canh or bistable model. We show that these systems can be controlled fulfilling the natural constraints if time is large enough. This is in contrast with the unconstrained case where parabolic systems can be controlled in an arbitrarily small time, thanks to the infinite velocity of propagation.

The method of proof combines various methods and, in particular, employs phase-plane analysis techniques allowing to build paths of steady-state solutions. The control strategy consists then in building trajectories of the time-evolving system in the vicinity of those paths.

We shall conclude our lecture with a number of challenging open problems.

This presentation is based on joint work with Jérôme Lohéac (CNRS-Nancy), Camille Pouchol and Emmanuel Trélat (LJLL-Sorbonne Univ.), Dario Pighin (UAM-Madrid) and Jiamin Zhu (Univ. Toulouse).

Our work was motivated by discussions with J.R. Uriarte from the Faculty of Economics of the University of Basque Country (UPV/EHU) who raised the problem of modeling and control of multi-linguism.

Contributed Talks

Mathematical Model for the Effect of Heat Transfer to Mass Concentration in a Stenosed Artery

Amira Husni Talib[1], Ilyani Abdullah[2], Nabilah Naser[3]
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A numerical study of mass and heat transfer in a stenosed arterial segment is investigated through this paper. The blood flow is treated as incompressible and two-dimensional power law fluid with a presence of cosine shaped stenosis. Heat transfer defines for heat moving from one body or substances to another. Meanwhile, mass transfer refers to the movement of low-density lipoprotein (LDL) in the artery and brings up to localization of stenosis. The influenced of heat transfer to mass concentration of LDL in blood flow is studied, with appropriate prescribed conditions. Marker and Cell (MAC) method is used to solve the problems. The graphical results are presented in more details. The mass concentration profiles are shown to be affected by heat variances, along with the present of stenosis at the arterial wall.

Reduction of the resonance error in numerical homogenization problems: a parabolic approach

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This work concerns the numerical homogenization of multiscale elliptic partial differential equations (PDEs) of the form

$$\begin{cases} -\nabla \cdot (A^{\varepsilon}(x)\nabla u^{\varepsilon}(x)) = f(x) & \text{in } \Omega \subset \mathbb{R}^d \\ u(x) = g(x), \quad \text{in } \partial\Omega, \end{cases} \tag{1}$$

where the coefficient A^{ε} is a symmetric, positive definite, uniformly bounded matrix function in $\mathbb{R}^{d\times d}$, and has microscopic variations of size $\varepsilon \ll |\Omega| = O(1)$. A direct numerical simulation of such a problem is prohibitively expensive since the ε-scale variations need to be resolved over the entire macroscopic domain Ω. As $\varepsilon \to 0$, the multiscale PDE (1) can be approximated by the homogenized PDE

$$\begin{cases} -\nabla \cdot (A^0(x)\nabla u^0(x)) = f(x) & \text{in } \Omega \\ u^0(x) = g(x), & \text{in } \partial\Omega, \end{cases} \tag{2}$$

where A^0 varies slowly over the domain Ω, and hence a numerical approximation to the homogenized solution u^0 can be obtained at a cost independent of ε. Explicit formulas for the homogenized coefficient A^0 are available only under restrictive structural assumptions on A^ε, e.g. periodic A^ε. To approximate u^0 in more general settings, multiscale methods are needed. Typical multiscale methods designed for approximating u^0 have two main components: a macro- and a micromodel. The micro problems are solved over small domains of size $O(\delta^d)$, where $\delta = O(\varepsilon)$, to upscale homogenized quantities, e.g. A^0, to the macroscale model. A common issue is then the presence of a resonance error of order ε/δ, which is due to imposing inaccurate boundary conditions in the micromodel. Reduction of this error has been a subject of interest over the last two decades. Here, we propose an approach based on parabolic micro problems, which improves the rate to higher orders.

The closest normal structured matrix

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For a given structured matrix $A \in \mathcal{S}$ we study the problem of finding its closest normal matrix with the same structure, i.e. solving $\min_{X\in\mathcal{S}\cap\mathcal{N}} \|A - X\|_F^2$. The structures of our interest are: Hamiltonian, skew-Hamiltonian, per-Hermitian, and perskew-Hermitian.

We show that solving this minimization problem is equivalent to finding the structure-preserving unitary transformation that maximizes the Frobenius norm of the "generalized diagonal" of A. For each of the matrix structures mentioned above we define the corresponding generalized diagonal form. Then we are solving the dual maximization problem with the objective function adapted to the specific structure. We propose a set of structure-preserving algorithms of the Jacobi type and prove their convergence to the stationary point of the associated objective function.

Speeding-up simultaneous reductions of several matrices to a condensed form

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We are concerned with the simultaneous orthogonal reductions of several matrices to a condensed form, based on the Givens rotations. The basic task of these algorithms is to reduce one matrix to Hessenberg or m-Hessenberg form, and the others to triangular form. Such condensed forms are suitable for solving multiple shifted systems $(\sigma E - A)X = B$, and for solving the generalized singular value problem $A^T Ax = \mu^2 B^T Bx$. At the beginning of the reduction algorithm, all matrices except one are reduced to the triangular form by QR factorization, and then the remaining matrix is reduced to the Hessenberg form while simultaneously preserving triangular form of the other matrices. The later reduction is performed by Givens rotations, which renders the whole algorithm very inefficient. We proposed several techniques for speeding-up applications of the rotations. One approach is based on blocking strategies on at least two levels, and the other approach exploits multithreading ability of modern CPUs, as well as parallel computing on GPU. Both approaches offer respectable speed-up factors. The optimal efficiency is obtained by combining the blocking strategy with parallel updates, and by overlapping the reduction step on the CPU with the compute-intensive updates based on matrix–matrix multiplications performed on the GPU.

Approximations by CCC–Schoenberg operators and contour stencils in image resampling

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Resampling of digital images is an essential part of image processing. The most efficient and sufficiently accurate image resampling techniques can produce ringing artifacts, due the oscillations of the approximations near sharp transitions of color when upsampling. Also, the methods, based on the tensor products of one dimensional methods, can produce stairlike lines (aliasing). To solve the first problem, we use shape preserving approximations by CCC–Schoenberg operators for the interpolation or histoploation process, applied dimension by dimension. The associated spline space is the space of variable degree polynomial splines. For the second one, we apply the contour stencils to approximate locally along the image edges. A special approximation, based on tensor product, is designed for each stencil. The local

approximations are calculated relatively simply, and than blended together to get the final approximation.

General quantum variational calculus

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The general quantum calculus was recently developed by Hamza *et al.* [2] and generalizes both the q-calculus [4] and the Hanh's calculus [1,3].

We develop a new variational calculus based in the general quantum difference operator recently introduced by Hamza et al. In particular, we obtain optimality conditions for generalized variational problems where the Lagrangian may depend on the endpoints conditions and a real parameter, for the basic and isoperimetric problems, with and without fixed boundary conditions. Our results provide a generalization to previous results obtained for the q- and Hahn-calculus.

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A Householder-based algorithm for Hessenberg-triangular reduction

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Reducing the matrix pair (A, B) to Hessenberg-triangular form is an important and time-consuming preprocessing step when computing eigenvalues and eigenvectors of the pencil $A - \lambda B$ by the QZ-algorithm. Current state-of-the-art algorithms for this reduction are based on Givens rotations, which limits the possibility of using efficient level 3 BLAS operations, as well as parallelization potential on modern CPUs. Both of these issues remain even with partial accumulation of Givens rotations, implemented, e.g., in LAPACK.

In this talk we present a novel approach for computing the Hessenberg-triangular reduction, which is based on using Householder reflectors. The key element in the new algorithm is the lesser known ability of Householder reflectors to zero-out elements in a matrix column even when applied from the right side of the matrix. The performance of the new reduction algorithm is boosted by blocking and other optimization techniques, all of which permit efficient use of level 3 BLAS operations. We also discuss measures necessary for ensuring numerical stability of the algorithm. While the development of a parallel version is future work, numerical experiments already show benefits of the Householder-based approach compared to Givens rotations in the multicore computing environment.

Extended Derrida-Lebowitz-Speer-Spohn equation

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Derrida-Lebowitz-Speer-Spohn (DLSS) equation [2] is a nonlinear fourth-order diffusion equation describing interface fluctuations between two phases of predominantly $+1$ and -1 spins in two-dimensional lattice spin system with north east center (NEC) majority rule (Toom model). The statistical model has been refined in [1], giving rise to the extended DLSS equation, which additionally includes a third-order term. Following ideas developed in [3], we will discuss the existence of global in time weak nonnegative solutions of the extended DLSS equation with periodic boundary conditions, as well as the long-time behaviour of solutions.

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Spectral analysis of thin domains in high-contrast regime

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In this work, we consider the resolvent problem for three-dimensional thin plates in linearized elasticity with high contrast in the coefficients, as the period ε and plate thickness h tend to zero simultaneously. In order to derive the limit models, we use two-scale convergence results adapted for dimension reduction. By dividing the problem into two invariant subspaces, we are able to prove different behaviours of eigenvalues for bending and membrane displacements.

This is a joint work with I. Velčić and J. Žubrinić.

Singular limits in fluid mechanics: "thin" and rotating fluids

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We study two problems related to singular limits in fluid mechanics.

First, we study the inviscid incompressible limits of the rotating compressible Navier–Stokes system for a barotropic fluid. We show that the limit system is represented by the rotating incompressible Euler equation on the whole space.

Second, we consider a heat conductive compressible Navier–Stokes–Fourier–Poisson system confined to a straight layer $\Omega_\epsilon = \omega \times (0, \epsilon)$, where ω is a 2–D domain. We show the convergence to the 2–D system when $\epsilon \to 0$. We study two different regimes in dependence on the behavior of the Froude number.

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Regularization of Inverse Scattering Problem in Ultrasound Tomography

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Most finite dimensional inverse problems that arise in applications are ill-posed. However, choosing a proper regularization algorithm depends on the problem itself. Here, our focus is on the ultrasound tomography, more precisely, a distorted Born iterative (DBI) method as a powerful way for reconstructing an ultrasound image. The inverse scattering part of DBI, denoted with $\mathbf{X}\mathbf{y} = \mathbf{b}$, is ill-posed, making it sensitive to errors. A regularization algorithm is necessary to ensure convergence of the DBI and produce a high quality ultrasound image.

We are presenting the advantages of algorithms that employ generalized singular value decomposition (GSVD) of matrix pair $(\mathbf{X}, \mathbf{L})$ over algorithms with SVD of matrix $\mathbf{X}$ for regularization of problem $\mathbf{X}\mathbf{y} = \mathbf{b}$ in DBI. The usage of matrix $\mathbf{L}$ provides additional regularization by smoothing the noise. Usually, a discrete version of first or second order derivative operator could be used. In addition, since the regularization properties of algorithms depend largely on the choice of good regularization parameter, we are proposing a new way for their determination suited for this problem. Numerical simulations for reconstruction of images using aforementioned methods are presented.

Fast Sweep Method For Computation of Isostables and Isochrons

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We propose a fast iterative algorithm for computation of isostables and isochrons for dynamical systems with stable limit cycles or fixed points in high dimensions. We formulate the problem as a solution to a first order static Hamilton–Jacobi equation with a constant source term and we solve the boundary problem using a Eulerian Fast Sweeping Method developed for this type of problems. We apply this method on several illustrative examples of dynamical systems and show that this is an efficient, accurate method which can be used in a data-driven setting.

Optimality criteria method for optimal design problems

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In optimal design problems the goal is to find the arrangement of given materials within the body which optimizes its properties with respect to some optimality criteria. The performance of the mixture is usually measured by an integral functional, while optimality of the mixture is achieved through minimization or maximization of this functional, under constraints on amount of materials and PDE constraints that underlay involved physics.

We consider multiple-state optimal design problems from conductivity point of view, where thermal (or electrical) conductivity is modeled with stationary diffusion equation and restrict ourselves to domains filled with two isotropic materials. Since the classical solution usually does not exist, we use relaxation by the homogenization method [2] in order to get a proper relaxation of the original problem.

One of numerical methods used for solving these problems is the optimality criteria method, an iterative method based on optimality conditions of the relaxed formulation. In the case of a single-state problem this method is described in [1], where it is also proved that it converges in the case of a self-adjoint optimization problems. Based on the optimality conditions derived in [1], a variant of optimality criteria method for multiple-state problems was introduced in [3]. It appears that this variant works properly for maximization of conic sum of energies, but fails for the minimization of the same functional.

We rewrite optimality conditions for relaxed problem and develop a variant of optimality criteria method suitable for energy minimization problems. We also prove convergence of this method in a special case when a number of states is less then the space dimension and in the spherically symmetric case. Presented method can be expanded to similar problems in the context of linearized elasticity.

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Spectral properties of the stochastic Koopman operator and its numerical approximations

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A generalization of the Koopman operator framework, originally developed for deterministic dynamical systems, to discrete and continuous time random dynamical systems results with the stochastic Koopman operators. The study of the spectral objects of these operators (Koopman eigenvalues, eigenfunctions and modes) could be useful in analysing the behaviour of the considered random dynamical system. We provide the results that characterize the spectrum and the eigenfunctions of the stochastic Koopman operators for the particular classes of random and stochastic dynamical systems. The numerical approximations of the eigenvalues and eigenfunctions of the stochastic Koopman operator are computed by using DMD RRR algorithm. Its behaviour in the stochastic framework is explored on several test examples. Furthermore, we introduce the isostables and isochrones associated to the random dynamical systems and compute their numerical approximations on the chosen example.

A shear flow problem for compressible viscous and heat conducting micropolar fluid

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We consider the non-stationary 3-D flow of a compressible and viscous heat-conducting micropolar fluid in the domain bounded by two parallel horizontal plates that present solid thermoinsulated walls. In the thermodynamical sense the fluid is perfect and polytropic, and we assume that the initial density and initial temperature are strictly positive.

In this work we present the existence and uniqueness results for corresponding one-dimensional problem in Lagrangian description with smooth enough initial data and non-homogeneous boundary data for velocity, as well as homogeneous boundary data for microrotation and heat flux.

Variations of the discrete empirical interpolation method

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Numerical simulations are a tool of trade in studying a great variety of complex physical phenomena in areas such as e.g. computational fluid mechanics, neuron modeling, or e.g. microchip design. High fidelity simulations require high resolution of the discretization, which leads to systems of ordinary/partial differential equations of ever-larger scale and complexity. Simulations in such large-scale settings are computationally intensive, in particular in the case of design optimization over a parameter space.

Model reduction approach is to create smaller, faster approximations to complex dynamical systems that still guarantee high fidelity. EIM (Barrault, Nguyen, Maday, Patera 2004)/DEIM (Chaturantabut, Sorensen 2010) is a powerful model reduction tool, in particular when combined with the Galerkin projection and the POD. To preserve physical properties of the reduced model, the POD projection must be with respect to a particular weighted inner product; it is then natural that the corresponding DEIM projection is weighted in the same way.

We present our recent results on the numerical implementation of the orthogonal DEIM (QDEIM, Drmač, Gugercin 2016), weighted DEIM (WDEIM, Drmač, Saibaba 2018), that can also be interpreted as a numerical implementation of the Generalized Empirical Interpolation Method (GEIM, Maday, Mula 2013) and the more general Parametrized-Background Data-Weak approach (PBDW, Maday, Patera, Penn, Yano, 2015). Further, we discuss new point selection strategies in the oversampled DEIM (Drmač, Gugercin, Peherstorfer 2018). Both the numerical analysis and the performance in simulations will be presented.

Velocity averaging and existence of solutions for degenerate parabolic equations

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We prove a velocity averaging result for the L^p-bounded, $p \geq 2$, sequence of solutions to degenerate dissipative transport equations. As a consequence, we prove existence of weak solution to degenerate parabolic equations with discontinuous flux.

In the proofs a new variant of H-measures (or microlocal defect measures) is introduced and used, which is adapted to equations that change type.

High order implicit relaxation schemes for nonlinear hyperbolic systems

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In this work we consider the time discretization of compressible fluid models which appear in gas dynamics, biology, astrophysics or plasma physics for Tokamaks. In general, for the hyperbolic system we use an explicit scheme in time. However, for some applications, the characteristic velocity of the fluid is very small compared to the fastest velocity speed. In this case, to filter the fast scales it is common to use an implicit scheme. The implicit schemes allows to filter the fast scale that we don't want to consider and choose a time step independent of the mesh step and adapted to the characteristic velocity of the fluid. The matrices induced by the discretization of the hyperbolic system are ill-conditioned in the regime considered and very hard to invert. In this work we propose an alternative method to the classical preconditioning, based on the BGK relaxation methods. The idea is, to propose a larger and simpler model (here a BGK model [1,2]) depending of the small parameter which approximate the original system. Designing an AP scheme based on splitting method [1] for the BGK model, stable without CFL condition, we obtain at the end a very simple method avoiding matrix inversion and unconditionally stable for the initial model. This method can approximate any hyperbolic models and can be generalized to treat models including additional small diffusion terms. After the presentation of the method we will show how obtained high-order schemes in time and space. To finish we will focus on two non trivial applications for the BGK relaxation methods: the low-mach regime for Euler equations and the parabolic models.

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Analysis of a nonlinear 3D fluid-mesh-shell interaction problem

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We study a nonlinear, unsteady, moving boundary fluid-structure interaction problem between an incompressible, viscous 3D fluid flow, a 2D linearly elastic Koiter shell, and an elastic 1D net of curved rods. This problem is motivated by studying fluid-structure interaction between blood flow through coronary arteries treated with metallic mesh-like devices called stents. The fluid flow, which is driven by the time-dependent dynamic pressure data, is governed by the Navier-Stokes equations, and the structure displacement is modeled by a system of linear Koiter shell equations allowing displacements in all three spatial directions. The fluid and the mesh-supported structure are coupled via the kinematic and dynamic coupling conditions describing continuity of velocity and balance of contact forces. We prove the existence of a weak solution to this nonlinear fluid-composite structure interaction problem using the Arbitrary Lagrangian Eulerian weak formulation and the time discretization via Lie operator splitting scheme.

Spectral analysis of an eigenvalue problem on a metric graph

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We present a spectral analysis of a second order constrained eigenvalue problem on a metric graph. The underlying operator is vector valued operator and it originates for the vibration analysis in the modelling of endovasular stents. We also study the dynamical problem for such a configuration and show that standard DAE integrators can solve the problem well. This is a joint work with V. Mehrmann and J. Tambaca.

Circle arc approximation by parametric polynomials

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We consider uniform approximation of a circle arc by a parametric polynomial curve. For low degree curves the approximant can be given in a closed form. For higher degrees a nonlinear equation needs to be solved. Error analysis is done, and application for fast evaluation of trigonometric functions is considered.

Composite elastic plate via general homogenization theory

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General, non-periodic homogenization theory is well developed for second order elliptic partial differential equations, where the key role plays the notion of H-convergence. It was introduced by Spagnolo through the concept of G-convergence (1968) for the symmetric case, and further generalized by Tartar (1975) and Murat and Tartar (1978) for non-symmetric coefficients under the name H-convergence. Some aspects for higher order elliptic problems were also considered by Zhikov, Kozlov, Oleinik and Ngoan (1979). Homogenization theory is probably the most successful approach for dealing with optimal design problems (in conductivity or linearized elasticity), that consists in arranging given materials such that obtained body satisfies some optimality criteria, which is mathematically usually expressed as minimization of some (integral) functional under some (PDE) constrains.

Motivated by a possible application of the homogenization theory in optimal design problems for elastic plates, we adapt the general homogenization theory for Kirchoff-Love elastic plate equation, which is a fourth order elliptic equation. In addition to the compactness result, we prove a number of properties of H-convergence, such as locality, irrelevance of the boundary conditions, corrector results, etc. Using this newly developed theory, we derive expressions for elastic coefficients of composite plate obtained by mixing two materials in thin layers (known as laminated materials), and for mixing two materials in low-contrast regime. Moreover, we also derive optimal bounds on the effective energy of a composite material, known as Hashin-Shtrikman bounds.

Analysis of a model for a magneto-viscoelastic material

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The talk is concerned with a mathematical model for a class of materials which possess the special property that they respond mechanically to applied magnetic fields and they change their magnetic properties in response to mechanical stresses. The model consists of a system of equations for the balance of momentum that is coupled with systems of equations describing the evolution of quantities related to elastic and magnetic properties of the material. The issue of existence as well as uniqueness of a solution to the system of partial differential equations under consideration will be discussed.

A Reduced Basis Approach for PDE problems with Parametric Geometry for Embedded Finite Element Methods

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We introduce and discuss some results related to unfitted finite element methods for parameterized partial differential equations enhanced by a reduced order method construction. A model order reduction technique is proposed to integrate the embedded boundary finite element methods. Results are validated numerically. This methodology which extracts an unfitted mesh Nitsche finite element method in reduced order proper orthogonal decomposition method is based on a background mesh and stationary Stokes flow systems are examined. This approach achievements are twofold. Firstly, we reduce much computational effort since the unfitted mesh method allows us to avoid remeshing when updating the parametric domain. Secondly, the proposed reduced order model technique gives implementation advantage considering geometrical parametrization. Computational are even exploited more efficiently since mesh is computed once and the transformation of each geometry to a reference geometry is not required. These combined advantages allow to solve many PDE problems more efficiently, and to provide the capability to find solutions in cases that could not be resolved in the past.

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Bressan's problem on mixing flows

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Around 2003 Alberto Bressan proposed an open problem on two incompressible fluids in a periodic container. Informally saying, it is conjectured that the minimal cost of mixing of the two fluids up to scale ε grows like $\log(1/\varepsilon)$ as $\varepsilon \to 0$. The most natural quantity representing this cost is the total variation of the time-dependent vector field v that causes the mixing. A weaker result, when the L^1 norm of ∇v is replaced by its L^p norm for $p > 1$, has been addressed several times in the existing literature. It was first established by Crippa and De Lellis (2006) by reducing it to certain estimates for the maximal function, and a similar technique was employed by Seis (2013). On the other hand, Hadžić, Seeger, Smart, and Street (2016) approached the L^p variant of the problem via estimates for multilinear singular integrals, and a modification of their idea was also used by Léger (2016). In this talk we discuss yet another approach to the aforementioned problem and its special cases using techniques from harmonic analysis.

Optimal design problems on annulus with classical solutions in 2D and 3D

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A multiple state optimal design problem for stationary diffusion equations with two isotropic phases is considered. Better conductivity is represented with β and worse with α. Distribution of phase α is denoted by characteristic function χ, so overall conductivity can be written by $A = \chi\alpha I + (1-\chi)\beta I$ and state equations are uniquely determined by temperatures $u_1, ..., u_m$:

$$\begin{cases} -\mathrm{div}(A\nabla u_i) = f_i \\ u_i \in H_0^1(\Omega) \end{cases} \qquad i = 1, ..., m.$$

Here, the right-hand sides $f_1, ..., f_m \in H^{-1}(\Omega)$ are given, and the aim is to maximize a conic sum of energies obtained for each state problem. Commonly, optimal design problems do not have solutions (such solutions are usually called *classical*).

Therefore, one needs to consider a proper relaxation of the original problem. A relaxation by the homogenization method was introduced and it is based on introduction of generalized composite materials, which are mixtures of original phases on a micro-scale. Such relaxed problems have solutions (we call them *relaxed* or *generalized* solutions).

It was showed that in case of spherical symmetry, it is possible to pass to a simpler relaxation given only in terms of local proportion of original phases. By analysing the optimality conditions we are able to show that in the case of annulus, the solution is also unique, classical and radial.

This rich class of analytical solutions for optimal design problems give opportunity to test different numerical methods. To demonstrate, method based on a shape derivative was implemented and tested in the `Freefem++`. Stable convergence to the optimal solutions was observed in both the 2D and 3D test examples.

Averaged controllability in a long time horizon

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We extend the recently introduced notion of averaged controllability for parameter dependent systems [1,2]. The goal is to design a control independent of the parameter that steers the averaged of the system to some prescribed value in time $T > 0$ but also keeps the averaged at this prescribed value for all times $t > T$. This new notion we address as *long-time averaged controllability*.

We consider finite dimensional systems and provide a necessary and sufficient condition for this property to hold. Once the condition is satisfied, one can apply a feedback control that keeps the average fixed during a given time period. We also address the L^2-norm optimality of such controls.

Relations between the introduced and previously existing different control notions of parameter dependent systems are discussed, accompanied by numerical examples.

This is a joint work with Jérôme Lohéac, University of Lorraine.

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3d structure – 2d plate interaction model

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In this paper we rigorously derive models for interaction of a linearized three-dimensional elastic structure with a thin elastic layer of thickness ε and of possibly different material attached to it. Furthermore the attached thin material is assumed to have the elasticity coefficients which are of order $1/\varepsilon^p$, for $p \geq 0$ with respect to the coefficients of the three-dimensional body. In the limit five different models are obtained with respect to different choices of p. Moreover a three-dimensional–two-dimensional model is proposed which has the same asymptotics as the original three-dimensional problem. This is convenient for applications since one do not have to decide in advance which limit model to use.
This is a joint work with J. Tambača.

Non-autonomous Koopman operator family spectrum

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Poincaré at the beginning of XX-th century, and then Carleman, Koopman and von Neumann in the 1920-is made their visionary contributions to the analysis of dynamical systems behavior through the analysis of the spectral properties of the associated Koopman operator. In this century the interest for the Koopman operator theory and applications is renewed thanks to the advances of the functional analysis as well as development of data-driven algorithms. Originally Koopman operators were aimed at ergodic theory of measure-preserving systems. Today applications to non-autonomous dynamical systems or dynamical systems in presence of uncertainty are of highest interest.

In this work we present results on the basic properties of the eigenvalues and eigenfunctions of the non-autonomous Koopman operators as well as the analysis of issues that arise when data-driven algorithms are applied to the evaluation of the non-autonomous Koopman eigenvalues and eigenvectors. The first data-driven approach is DMD application to moving windows of snapshots. In such approach all DMD methods manifest significant errors and we analyze and prove the structure of these errors. The second data-driven approach is DMD application to large Hankel matrices of snapshots. In this approach we investigate the relation between the non-autonomous Koopman operator eigenvalues and eigenfunctions and the eigenvalues and eigenfunctions of the underlying extended autonomous dynamical system. We illustrate the results of our analysis on several synthetic test-examples.

A second Noether-type theorem for delayed higher-order variational problems of Herglotz

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It is well-known that the classical variational principle is a powerful tool in several disciplines such as physics, engineering and mathematics. However, the classical variational principle cannot describe many important physical processes. One attempt to solve this limitation was done in 1930 by Gustav Herglotz [1] with the study of the following problem: determine trajectories $x \in C^1([a,b];\mathbb{R}^m)$ and $z \in C^1([a,b];\mathbb{R})$ that minimize the final value of the function z, where $\dot{z}(t) = L\left(t, x(t), \dot{x}(t), z(t)\right)$, subject to $z(a) = \gamma$, $x(a) = \alpha$ and $x(b) = \beta$, for fixed $\gamma \in \mathbb{R}$, $\alpha, \beta \in \mathbb{R}^m$, and the Lagrangian L satisfies some appropriate regularity assumptions.

The main goal of this talk is to present Noether currents for higher-order problems of Herglotz type with time delay [2]. Our work is related with the second Noether theorem for optimal control in the sense of [3], and is particularly useful because provides necessary conditions for the search of extremals. The proof is based on the idea of rewriting the higher-order delayed generalized variational problem as a first-order optimal control problem without time delay. As a corollary of our main result, we obtain a new result for delayed classical problems of the calculus of variations.

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Exponentially fitted difference schemes on adapted meshes

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We consider two-point singularly perturbed boundary value problem of the form:

$$\varepsilon y'' + by' + cy = f, \quad y(0) = \alpha, \quad y(1) = \beta,$$

where ε is a small parameter ($0 < \varepsilon \ll 1$) and function c satisfies $c < 0$.

Solution of the above differential equation exhibits so called boundary layer phenomena, i.e., exponential behaviour at the one end of the interval (side depends on the sign of function b). Classical approach to this boundary value problem fails since an argument 'a method converges for sufficiently small meshwidth h' implies that h is of the same size as parameter ε what is unacceptable in a practice.

Common requirement for numerical methods applied to singularly perturbed problems is ε-uniform convergence. A method is ε-uniform convergent if there exist constants C and m, independent on ε, such that exact solution u of the problem and its approximation at mesh points u_i satisfy

$$\max_{\varepsilon\in[0,1]} |u(x_i) - u_i| \le Ch^m$$

for all i.

One approach to avoid this problem is to use a mesh that is dense in the boundary layer. Many standard methods appear to be uniform convergent in this case. Another approach, discussed here, is to use exponentially fitted scheme, i.e., scheme that gives exact solution if it is an exponential function. Such schemes are not ε-uniform convergent [7].

However, they satisfy

$$\max_{i} |u(x_i) - u_i| \le Ch^m$$

when $h \ge 4(m-2)\varepsilon \ln(1/\varepsilon)/b_{\min}$ for the method of order m. Although the given bound on h is not a great restriction in practice, we construct an adapted mesh, dense in boundary layer, that guaranties the same bound in a case of smaller h.

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Asymptotic analysis of the viscous flow through a pipe and the derivation of the Darcy-Weisbach law

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Darcy-Weisbach formula is used to compute the pressure drop of the fluid in the pipe, due to the friction against the wall. Because of its simplicity, the Darcy-Weisbach formula become widely accepted by engineers and is used for laminar as well as the turbulent flows through pipes, once the method to compute the mysterious friction coefficient was derived. Particularly in the second half of the 20-th century. Formula is empiric and our goal is to derive it from the basic conservation law, via rigorous asymptotic analysis. We consider the case of the laminar flow but with significant Reynolds number. In case of the perfectly smooth pipe, the situation is trivial, as the Navier-Stokes system can be solved explicitly via the Poiseuille formula leading to the friction coefficient in the form 64/Re. For the rough pipe the situation is more complicated and some effects of the roughness appear in the friction coefficient. We start from the Navier-Stokes system in the pipe with periodically corrugated wall and derive an asymptotic expansion for the pressure and for the velocity. We use the homogenization techniques and the boundary layer analysis. The approximation derived by formal analysis is then justified by rigorous error estimate in the norm of the appropriate Sobolev space, using the energy formulation and classical a priori estimates for the Navier-Stokes system. Our method leads to the formula for the friction coefficient. The formula involves resolution of the appropriate boundary layer problems, namely the boundary value problems for the Stokes system in an infinite band, that needs to be done numerically. However, theoretical analysis characterising their nature can be done without solving them.

Cosine-Sine Decompositions
(Some Open Problems and Some Applications)

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The cosine-sine decomposition (CSD) of orthogonal matrices is a widely used tool for theoretical and computational purposes in numerical linear algebra. This short report addresses several open problems that are linked to the CSD. In particular, we would like to shed more light on challenges that are met in computing the CSD when the sines and cosines from that decomposition are multiple or very close.

Next, we show how the CSD can be used to (try to) solve the following problem: how to diagonalize a symmetric matrix of small dimension with just several plane rotations? In particular, the symmetric matrix of order 3 (4, 5, 6) can be diagonalized using 3 (6, 10, 15) rotations. How to find those rotations? So far, only the problem with the smallest dimension 3 has been solved, using quaternions. Our goal is to solve at least some of those problems more directly.

Similar open problems are also linked to the hyperbolic CSD of J-orthogonal matrices.

New algorithms for detecting a hyperbolic quadratic eigenvalue problem

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The quadratic eigenvalue problem (QEP) is to find scalars λ and nonzero vectors x such that $\mathbf{Q}(\lambda)x := (\lambda^2 M + \lambda D + K)x = 0$ holds for the given matrices M, D and K. In this talk we consider a QEP where M, D, K are Hermitian matrices with M positive definite. Additionally, if there exists a real number λ_0 such that the matrix $\mathbf{Q}(\lambda_0)$ is negative definite, then the given QEP is called *hyperbolic*. We are interested in detecting if a given QEP is hyperbolic or not. Although there exist many algorithms for detecting the hyperbolicity, most of them are not suited for large QEPs.

We propose a new basic subspace algorithm for detecting large hyperbolic QEPs. Our algorithm is based on iterative testing of small compressed QEPs formed by using search subspaces of small dimensions. How to choose search subspaces is a non trivial question. Therefore, we propose a very simple type of search subspaces and get specialized algorithms for detecting hyperbolic QEPs. These algorithms are based on Locally Optimal Block (Preconditioned) Extended Conjugate Gradient (LOB(P)eCG) Methods for computing extremal eigenpairs of large hyperbolic QEPs, proposed in [1] and have a monotonicity property based on Cauchy-type interlacing inequalities [3]. If a small nonhyperbolic compressed QEP is find, the algorithms end with the conclusion that the given large QEP is nonhyperbolic. The algorithms find a number λ_0 if the given large QEP is hyperbolic. Our algorithms can be easily adapted to detect a large *overdamped* QEP (meaning, it is hyperbolic with D positive definite and K positive semidefinite). Numerical experiments demonstrate the efficiency of our specialized algorithms.

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Eigensubspace perturbation bounds for quadratic eigenvalue problem

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We present new relative perturbation bounds for the eigensubspaces for quadratic eigenvalue problem $\lambda^2 Mx + \lambda Cx + Kx = 0$, where M and K are nonsingular Hermitian and C is any Hermitian matrix. First, we derive the $\sin\Theta$ type theorems for the eigensubspaces of the regular matrix pairs (A, B), where both A and B are Hermitian matrices. Using a proper linearization and new relative perturbation bounds for regular matrix pairs (A, B), we develop corresponding $\sin\Theta$ type theorems for the eigensubspaces for the considered regular quadratic eigenvalue problem. Our bound can be applied to the gyroscopic systems which will be also shown. The obtained bounds will be illustrated by numerical examples.

Mathematical Model for Drug Release from a Swelling Device with Initial Burst Effect

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In practice, a new developed drug delivery device will undergo an in vitro experiment to determine the device's release profile. Typically, the drug release data is then fitted to certain mathematical formula such as zero-order, first order, Korsmeyer-Peppas, Higuchi model and many more to determine the release rate and mechanism. However none of these mathematical model take initial burst phenomenon into account although the phenomenon frequently happen in the experiment. Initial burst phenomenon is a situation where the initial relase rate is higher than the overall release rate. Therefore, in this research, we propose a mathematical formula to imitate the drug release profile from a swelling device with considering the initial burst effect.

A mathematical model for drug release from a swelling device is developed using advection-diffusion equation. Landau transformation and the method of Separation of Variable are employed to analytically solve the model. By having the solution of this basic model, we extend the model by adding the initial burst effect. The whole release profiles are divided into two phases where the first phase represents the initial burst release. Each phase has different diffusion coefficient. Finally, the analytical solution is utilized to estimate the diffusion coefficients and burst time of tested drug delivery devices using unconstrained optimization technique.

Optimal passive control of vibrational systems using mixed performance measures

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We will present new performance measures for vibrational systems based on the H_2 norm of linear control systems. Examples, both theoretical and concrete, will be given showing how these performance measures stack up against standard ones when used as an optimization criterion for the optimal damping of vibrational systems.

The talk is based on a joint work with Zoran Tomljanović and Ninoslav Truhar.

A numerical analytic continuation and its application to Fourier transform

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In this paper, we present a numerical method of analytic continuation and its application to numerical Fourier transform. We consider an analytic function given in a power series $f(z) = \sum_{n=0}^{\infty} c_n z^n$. We propose to compute the analytic continuation of $f(z)$ by transforming it into a continued fraction

$$f(z) = \frac{a_0}{1+} \frac{a_1 z}{1} + \frac{a_2 z}{1} + \cdots$$

and evaluating it. In general, the convergence region of the continued fraction is wider than that of the power series and, therefore, we can expect that the continued fraction numerically gives an analytic continuation of the power series. Numerical examples show that the presented method works well. The coefficients a_n of the continued fraction are obtained from those of the power series c_n by the quotient difference method [2], where we use multiple precision arithmetic because the method is numerically unstable.

We also apply this method of analytic continuation to the computation of the Fourier transform

$$\mathcal{F}[f](\xi) = \int_{-\infty}^{\infty} f(x) \mathrm{e}^{-2\pi \mathrm{i} \xi x} \mathrm{d}x.$$

The computation of the Fourier transform needs a numerical integration of an oscillatory function over the infinite interval, which is difficult if the integrand decays slowly at infinity. In hyperfunction theory [1], the Fourier transform is given by the difference of the values on $\mathbb{R}$ of two analytic functions. We compute these analytic functions and, then, we obtain the Fourier transform by extending them analytically onto the real axis by the numerical analytic continuation given above. Some examples show the efficiency of the presented method.

Acknowledgements This work is supported by JSPS KAKENHI Grant Number JP16K05267.

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Existence of local extrema of positive solutions of nonlinear second-order ode's and application

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We propose some conditions on the coefficients in arbitrarily given interval (a, b) such that every positive solution $x = x(t)$ of the nonlinear equation

$$(r(t)x'(t))' + p(t)f(x) + \sum_{j=1}^{m} q_j(t)|x|^{\alpha_j - 1}x = e(t)$$

has a local maximum attained in (a, b). These conditions are expressed in the term of the Rayleigh quotient associated to the linear eigenvalue problem on (a, b) with the Dirichlet boundary conditions. In some cases of the nonlinear term $f(x)$, the main result can verify the non-monotonic behaviour in some known mathematical models in applied sciences, which have been already numerically predicted. It continues the work on this subject already published in the papers presented below (chronological order) and references therein.

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Effects of small boundary perturbation on the porous medium flow

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It is well-known that only a limited number of the fluid flow problems can be solved (or approximated) by the solutions in the explicit form. To derive such solutions, we usually need to start with (over)simplified mathematical models and consider ideal geometries on the flow domains with no distortions introduced. However, in practice, the boundary of the fluid domain can contain various small irregularities (rugosities, dents, etc.) being far from the ideal one. Such problems are challenging from the mathematical point of view and, in most cases, can be treated only numerically. The analytical treatments are rare because introducing the small parameter as the perturbation quantity in the domain boundary forces us to perform tedious change of variables. Having this in mind, our goal is to present recent analytical results on the effects of a slightly perturbed boundary on the fluid flow through a channel filled with a porous medium. We start from a rectangular domain and then perturb the upper part of its boundary by the product of the small parameter ε and arbitrary smooth function. The porous medium flow is described by the Darcy-Brinkman model which can handle the presence of a boundary on which the no-slip condition for the velocity is imposed. Using asymptotic analysis with respect to ε, we formally derive the effective model in the form of the explicit formulae for the velocity and pressure. The obtained asymptotic approximation clearly shows the nonlocal effects of the small boundary perturbation. The error analysis is also conducted providing the order of accuracy of the asymptotic solution.

Recompression of Hadamard Products of Tensors in Tucker Format

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The Hadamard product features prominently in tensor-based algorithms in scientific computing and data analysis. Due to its tendency to significantly increase ranks, the Hadamard product can represent a major computational obstacle in algorithms based on low-rank tensor representations. It is therefore of interest to develop recompression techniques that mitigate the effects of this rank increase. In this work, we investigate such techniques for the case of the Tucker format, which is well suited for tensors of low order and small to moderate multilinear ranks. Fast algorithms are attained by combining iterative methods, such as the Lanczos method and randomized algorithms, with fast matrix-vector products that exploit the structure of Hadamard products. The resulting complexity reduction is particularly relevant for tensors featuring large mode sizes I and small to moderate multilinear ranks R. To implement our algorithms, we have created a new Julia library for tensors in Tucker format.

Perturbation Bounds for Parameter Dependent Quadratic Eigenvalue Problem

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We consider a quadratic eigenvalue problem (QEP):

$$(\lambda^2 M + \lambda D + K)x = 0, \tag{3}$$

where matrices M and K are Hermitian semidefinite and at least one of them is positive definite.

The most widely used approach for solving the polynomial (which includes QEP) eigenvalue problem is to linearize in order to produce a larger order pencil, whose eigensystem can be found by any method for generalized eigenproblems. This approach has been used, for example, in [3], [2].

To avoid linearizion (or simultaneous diagonalization of M and K, which is sometimes the preprocessing step, as in [3]), we propose two different types of bounds, the first is a simple first order approximation of function of several variables while the second one considers structured perturbation.

Thus, let $X = [x_1, \ldots, x_n]$ be a nonsingular matrix which contains n linearly independent right eigenvectors, and similarly, let $Y = [y_1, \ldots, y_n]$ be nonsingular matrix which contains n linearly independent left eigenvectors of QEP (3).

The corresponding perturbed QEP (3) is given by:

$$(\widetilde{\lambda}^2(M+\delta M)+\widetilde{\lambda}(D+\delta D)+K+\delta K)\widetilde{x}=0, \tag{4}$$

where $(\widetilde{\lambda}_i, \widetilde{x}_i)$ is perturbed eigenpair of (4).

The first bound is the upper bound for the first order approximation, based on Taylor's theorem, for the eigenvalues and the corresponding left and right eigenvectors of the following QEP

$$(\lambda^2(v)M(v)+\lambda(v)D(v)+K(v))x(v)=0, \tag{5}$$

where all three matrices M, D and K depend on $v=[v_1,\ldots,v_s]\in\mathbb{R}^s$. In this way we are able to efficiently calculate approximation of perturbed eigenvalues.

The second bound is of the following form

$$|y_i^*(M+T_{ij})\widetilde{x}_j| \le \frac{\|y_i^*\delta M\|}{RG1}+\frac{\|y_i^*\delta C\|}{RG2}+\frac{\|y_i^*\delta K\|}{RG3},$$

where y_i is i-th left eigenvector and

$$\begin{aligned}
T_{ij} &= \frac{D_{ij}}{\widetilde{\lambda}_j+\lambda_i}, && (6)\\
RG1 &= \min_{\substack{i=i_1,\ldots,i_p\\ j=j_1,\ldots,j_q\\ i\neq j}} \frac{|\lambda_i^2-\widetilde{\lambda}_j^2|}{|\widetilde{\lambda}_j^2|}, && (7)\\
RG2 &= \min_{\substack{i=i_1,\ldots,i_p\\ j=j_1,\ldots,j_q\\ i\neq j}} \frac{|\lambda_i^2-\widetilde{\lambda}_j^2|}{|\widetilde{\lambda}_j|}, && (8)\\
RG3 &= \min_{\substack{i=i_1,\ldots,i_p\\ j=j_1,\ldots,j_q\\ i\neq j}} |\lambda_i^2-\widetilde{\lambda}_j^2|. && (9)
\end{aligned}$$

Here we also have the bounds for relative gaps (7),(8),(9), which can be efficiently calculated.

As presented in [4], the derivatives of eigenvalues and eigenvectors with respect to v_i can be calculated, more or less efficiently, depending on the multiplicity of eigenvalues. Using these results, we will estimate the quality of the approximation for the eigenvalues and eigenvectors based on the algorithm from [3]. We will use optimization methods presented in [5], together with given approximations and upper bounds, in order to optimize damping efficiently.

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The transport speed and optimal work in pulsating Frenkel-Kontorova models

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We consider a generalized one-dimensional chain in a periodic potential (the Frenkel-Kontorova model), with dissipative, pulsating (or ratchet) dynamics as a model of transport when the average force on the system is zero. We find lower bounds on the transport speed under mild assumptions on the asymmetry and steepness of the site potential. Physically relevant applications include explicit estimates of the pulse frequencies which maximize transport. The bounds explicitly depend on the pulse period and number-theoretical properties of the mean spacing. The main tool is the study of time evolution of spatially invariant measures in the space of measures equipped with the L^1-Wasserstein metric.

Weak-strong uniqueness property for 3D fluid-rigid body interaction problem

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We consider incompressible Navier-Stokes equations coupled with a system of ordinary diferential equations of momentum conservation laws describing the motion of the rigid body in a fluid, filling a 3D bounded domain. We assume no-slip condition on the boundary. Weak-strong uniqueness property says that strong solutions, i.e. weak solutions that possess extra regularity, are unique in the larger class of weak solutions. The goal of this work is to extend the classical weak-strong uniqueness result for the Navier-Stokes equations, which requires only $L^p - L^q$ regularity of a strong solution, to the described fluid-rigid body interaction problem.

Rigorous derivation of a higher–order model describing the nonsteady flow of a micropolar fluid in a thin pipe

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This is a joint work with Prof. Michal Beneš and Prof. Igor Pažanin. In this talk, we will present a rigorous derivation of the model describing the nonsteady flow of a micropolar fluid through a pipe with arbitrary cross–section. Based on the existence and uniqueness result for the nonstationary micropolar Poiseuille solution in an infinite cylinder, we will construct a complete asymptotic expansion of the solution up to an arbitrary order, study the boundary layers in time and justify the usage of the formally derived asymptotic model via error estimate. Finally, we will present some numerical examples in the case of a circular cross–section and external force functions depending only on time.

Banking risk under epidemiological point of view

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A set of ordinary differential equations is presented, as an epidemiological model, using as application the banking risk and the network between banks. The contagion effect in the network is tested by implementing an epidemiological model, comprising a number of European countries and using bilateral data on foreign claims between them. Some numerical simulations based on the Pontryagin's Maximum Principle (indirect methods) and methods that treat the Optimal Control problem as a non-linear constrained optimization problem (direct methods) are tested and compared, using different numerical solvers.

On the Motion of Several Disks in an Unbounded Viscous Incompressible Fluid

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In this talk, we will present a recent result on fluid solid interaction problem. We consider the system formed by the incompressible Navier Stokes equations coupled with Newton's laws to describe the motion of a finite number of homogeneous rigid disks within a viscous homogeneous incompressible fluid in the whole space $\mathbb{R}^2$. The motion of the rigid bodies inside the fluid makes the fluid domain time dependent and unknown a priori. First, we generalize the existence and uniqueness of strong solutions result of the considered system in the case of a single rigid body moving in a bounded cavity in [3], and then by careful analysis of how elliptic estimates for the Stokes operator depend on the geometry of the fluid domain, we extend these solutions up to collision. Finally, we prove contact between rigid bodies cannot occur for almost arbitrary configurations by studying the distance between solids by a multiplier approach [1]. This talk is based on the results of the preprint [2].

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Stability and optimal control of compartmental models

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In this talk we propose mathematical compartmental models given by systems of ordinary differential equations (ODE), delayed differential equations (DDE) and fractional differential equations (FDE). For these models, the local, global and uniform stability of the equilibrium points is proved [1, 2, 3]. We formulate and solve optimal control problems associated to the models given by ODE's and DDE's systems. The theoretical results are illustrated through numerical simulations [2].

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An algorithm for the solution of quartic eigenvalue problems

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Quartic eigenvalue problem $(\lambda^4 A + \lambda^3 B + \lambda^2 C + \lambda D + E)x = 0$ appears in a variety of applications, e.g. calibration of the central catadioptric vision system and spatial stability analysis of the Orr Sommerfeld equation.
The standard approach for solving the polynomial eigenvalue problem is to linearize it, and then use the QZ algorithm to solve corresponding generalized eigenvalue problem. However, De Terán, Dopico and Mackey developed equivalence relation, so called quadratification, that converts quartic eigenvalue problem into an equivalent quadratic eigenvalue problem. Hammarling, Munro, and Tisseur developed the algorithm for the complete solution of this problem: `quadeig`.

We analyse numerical properties of the `quadeig` algorithm when used for solving the quartic eigenvalue problem. We propose modifications in two key segments of the algorithm: scaling and deflation of zero and infinite eigenvalues. Specifically, we use the structure of the quadratification for rank determination of coefficient matrices, which is the main part of deflation process. In addition, we determine the test for the existence of Jordan blocks for infinite and zero eigenvalues in terms of the original quartic problem.
Finally, we provide numerical examples to illustrate the power of the proposed algorithm.

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A new Naghdi type shell model

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A shell model is a two-dimensional model of a three-dimensional elastic body which is thin in one direction. There are several linear shell models in the mathematical literature that are rigorously justified starting from the linearized 3d elasticity. Examples are the membrane shell models, the flexural shell model and the Koiter shell model. Their application depends on the particular geometry of the shell's middle surface and the boundary condition which allow or disallow inextensional displacements.

In this talk a Naghdi type shell model will be presented and related to the classical models. This new model is given in terms of a displacement vector and the vector of infinitesimal rotation of the cross-section of the shell, both being in H^1. It unites different possible behaviors of the shell, it is applicable for all geometries and all boundary conditions, no complicated differential geometry is necessary for the analysis of the model and the model is also well formulated for geometries of the middle surface of the shell with corners. Asymptotically, with respect to the thickness of the shell, it behaves as the classical membrane, generalized membrane and flexural shell model depending on the particular regime and thus is a good approximation of the three-dimensional elasticity. Further, the solutions of the model continuously depend on the geometry.

Calculus of variations with combined variable order derivatives

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In this talk we present two generalizations of fractional variational problems by considering higher–order derivatives and a state time delay. For both problems, we establish several necessary optimality conditions for functionals containing a combined Caputo fractional derivative of variable fractional order, subject to boundary conditions at the initial time $t = a$. Because the endpoint is considered to be free, we also deduce associated transversality conditions.

Upper and lower bounds for sines of canonical angles between eigenspaces for regular Hermitian matrix pairs

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We present an upper and a lower bound for the the Frobenius norm of the matrix $\sin\Theta$, of the sines of canonical angles between unperturbed and perturbed eigenspaces of a regular generalized Hermitian eigenvalue problem $Ax = \lambda Bx$ where A and B are Hermitian $n \times n$ matrices, under a feasible non Hermitian perturbation. As one application of the obtained bounds we present the corresponding upper and the lower bounds for eigenspaces of a matrix pair (A, B) obtained by a linearization of regular quadratic eigenvalue problem $(\lambda^2 M + \lambda D + K)\, u = 0$, where M is positive definite and D and K are semidefinite.

We also apply obtained upper and lower bounds to the important problem which considers the influence of adding a damping on mechanical systems. The new results show that for certain additional damping the upper bound can be too pessimistic, but the lower bound can reflect a behaviour of considered eigenspaces properly.

Computational modeling of shape memory materials

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A sharp-interface model describing static equilibrium configurations of shape mory alloys introduced in [1] is extended to a quasistatic situation and computationally tested. Elastic properties of variants of martensite and the austenite are described by polyconvex energy density functions. Volume fractions of prticular variants are modeled by a map of bounded variation. Additionally, energy stored in martensite-martensite and austenite-martensite interfaces is measured by a interface-polyconvex function. It is assumed that transformations between material variants are accompanied by energy dissipation which, in our case, is positively and one-homogeneous giving rise to a rate-independent model. Two-dimensional computational examples are presented. This is a joint work with Miroslav Frost and Martin Kružík (both Prague).

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Uncertainty principles and null-controllability of the heat equation on bounded and unbounded domains

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In the talk I discuss several uncertainty relations for functions in spectral subspaces of Schrödinger operators, which can be formulated as (stationary) quantitative observability estimates. Of particular interest are unbounded domains or (a sequence of) bounded domains, with multi-scale structure and large diameter. The stationary observability estimates can be turned into control cost estimates for the heat equation, implying in particular null-controlability. The interesting question in the context of unbounded domains is: Which geometric properties needs a observability set to have in order to ensure null-controlability and efficient control cost estimates?
The talk is based on two joint projects, one with I. Nakić, M. Täufer, and M. Tautenhahn, the other with M. Egidi.

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Fractal properties of a class of polynomial planar systems having degenerate foci

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We study a class of polynomial planar systems with singularity of degenerate focus type without characteristic directions. This class is obtained using a natural transformation of a class of systems having weak foci, which is related to the normal form for the Hopf-Takens bifurcation. The class is given by

$$\begin{aligned} \dot{x} &= -y^{2n-1} \pm x^n y^{n-1}(x^{2n}+y^{2n})^k \\ \dot{y} &= \quad x^{2n-1} \pm x^{n-1} y^n (x^{2n}+y^{2n})^k, \end{aligned}$$

where parameters $k, n \in \mathbb{N}$.

For this class we compute the box dimension of any spiral trajectory Γ,

$$\dim_B \Gamma = 2\left(1 - \frac{1}{2nk+1}\right)$$

and show the connection to cyclicity under a perturbation.

This work is a continuation of the previous work done by Darko Žubrinić and Vesna Županović, regarding fractal analysis of spiral trajectories of planar vector fields.

Defect distributions related to weakly convergent sequences in Bessel type spaces

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Microlocal defect distributions (also called H-distributions) are introduced by Antonić and Mitrović (2011) for weakly convergent sequences in dual pair of $L^p - L^q$ spaces. They are extension of H-measures, introduced by Tartar (1990) and Gérard (1991) under the name of microlocal defect measures. Motivation for introducing these objects is the analysis of existence of solution of partial differential equation with a sequence of weak solutions which corresponds to the sequence of approximating equations.

We construct H-distributions for weakly convergent sequences in dual Bessel potential spaces, $H^q_s - H^p_{-s}$, $s \in \mathbb{R}, 1 < p < \infty$. Further we consider microlocal defect distributions associated to a weakly convergent sequences in $H^{-s,p}_\Lambda - H^{s,q}_\Lambda$ spaces using pseudo-differential operators with the symbols in $(s_\Lambda^{m,N+1})_0$, which correspond to a weight function Λ (cf. Nicola-Rodino 2010). Results are applied to partial differential equations with symbols related to weights of the type Λ.

Some remarks on the homogenization of immiscible incompressible two-phase flow

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We prove the homogenization result for incompressible two-phase flow in double porosity media. The fractured porous medium consists of periodically repeating homogeneous matrix blocks of ordinary porous media and fractures, with the absolute permeability being discontinuous at the boundary between the two media. The starting microscopic model consists of the mass conservation law along with the standard Darcy-Muskat law, for both fluids, and it is written in terms of the phase formulation. We consider a domain made up of several zones with different characteristics: porosity, absolute permeability, relative permeabilities and capillary pressure curves. The model involves highly oscillatory characteristics and internal nonlinear interface conditions. Under some realistic assumptions on the data, we prove the convergence of the solutions and derive the macroscopic models corresponding to various range of contrast by using the two-scale convergence method combined with the dilatation technique. The results improve upon previously derived effective models to highly heterogeneous porous media with discontinuous capillary pressures. This is a joint work with Brahim Amaziane (University of Pau and Pays de l'Adour), Mladen Jurak (University of Zagreb) and Leonid Pankratov (MIPT).

Sequential Predictors under Time-Varying Feedback and Measurement Delays and Sampling

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This talk will present the speaker's approach to delay compensation based on sequential predictors, which can compensate for arbitrarily long input delays, including in nonlinear control systems, or systems with time-varying delays as well as the effects of aperiodic sampling. The work was a joint project with Professor Michael Malisoff of Louisiana State University.

An existence result for a system modeling two-phase two-componet flow in porous medium in low solubility regime

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We present the existence result of a weak solution of the initial boundary value problem representing two-phase two-component fluid flow in porous media. The fluid considered consists of two phases: liquid and gas. The fluid is also a mixture of two components: an incompressible liquid component that does not evaporate and a gas component weakly soluble in the liquid. Low solubility of the gas component in the liquid phase allows us to treat the model without any unphysical assumptions on the diffusive parts.

The existence of the weak solution is proven by regularization of the system with a small parameter η and a time discretization in order to obtain the sequence of elliptic problems. The existence theorem for the elliptic problems is proved by Schauder fixed point theorem, using additional regularizations and suitable test functions in order to obtain energy estimates. By passing to the limit in time discretization parameter and regularization parameter η, we obtain a solution of the introduced problem.

A biodegradable elastic stent model

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A stent is a mesh, usually metallic, that is used to treat a narrow, injured or closed part of an artery to open and restore normal blood flow. In this talk we shall discuss and analyze how a one-dimensional model of biodegradable elastic stent is derived. The model is given as a nonlinear system of ordinary differential equations on a graph defined by geometry of stent struts. The unknowns in the problem are displacement of the middle curve of the struts, infinitesimal rotation of the cross–sections of stent struts, contact couples and contact forces at struts and a function describing degradation of the stent. The model is based on the one-dimensional model of biodegradable elastic curved rod model by Tambača and Žugec (One-dimensional quasistatic model of biodegradable elastic curved rods, *Zeitschrift für Angewandte Mathematik und Physik* 2015; 66(5): 2759–2785) and the ideas from the one-dimensional elastic stent modelling by Tambača et al. (Mathematical modeling of vascular stents, *SIAM Journal on Applied Mathematics* 2010; 70(6): 1922-1952) used to formulate contact conditions at vertices. We prove the existence and uniqueness result for the model.

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