# ApplMath$^{20}$

Tenth Conference on Applied
Mathematics and Scientific Computing

**14-18 September 2020, Brijuni, Croatia**

# Contents

# Main information

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## List of participants

1. Nenad Antonić, *University of Zagreb, Faculty of Science, Department of Mathematics*
2. Rokas Astrauskas, *Vilnius University*
3. Angela Bašić-Šiško, *University of Rijeka, Faculty of Engineering*
4. Erna Begović Kovač, *University of Zagreb*
5. Andreea Bejenaru, *University Politehnica of Bucharest, Romania*
6. Nela Bosner, *University of Zagreb, Faculty of Science, Department of Mathematics*
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21. Giovanni Gravina, *Charles University*
22. Matko Grbac, *University of Zagreb, Faculty of Science, Department of Mathematics*
23. Luka Grubišić, *University of Zagreb, Faculty of Science, Department of Mathematics*
24. Rupali Gupta, *Sardar Vallabhbhai National institute of Technology Surat, Surat*
25. Jelena Jankov, *Department of Mathematics, University of Osijek*
26. Martin Kalousek, *Institute of Mathematics of the Czech Academy of Sciences*
27. Malte Kampschulte, *Charles University Prague*
28. Vjekoslav Kovač, *University of Zagreb, Faculty of Science, Department of Mathematics*
29. Petar Kunštek, *University of Zagreb, Faculty of Science, Department of Mathematics*
30. Matko Ljulj, *University of Zagreb, Faculty of Science, Department of Mathematics*
31. Vaclav Macha, *Institute of Mathematics of the Czech Academy of Sciences*
32. Senka Maćešić, *University of Rijeka*
33. Eduard Marušić-Paloka, *University of Zagreb, Faculty of Science, Department of Mathematics*
34. Marija Miloloža Pandur, *Department of mathematics, J. J. Strossmayer University of Osijek*
35. Suzana Miodragović, *Department of mathematics, J. J. Strossmayer University of Osijek*

36. Petar Mlinarić, *Max Planck Institute for Dynamics of Complex Technical Systems*

37. Boris Muha, *University of Zagreb, Faculty of Science, Department of Mathematics*

38. Sarka Necasova, *Institute of Mathematics, Academy of Sciences of the Czech Republic*

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53. Ivana Šain Glibić, *University of Zagreb, Faculty of Science, Department of Mathematics*

54. Josip Tambača, *University of Zagreb, Faculty of Science, Department of Mathematics*

55. Zoran Tomljanović, *Department of Mathematics, Josip Juraj Strossmayer University of Osijek, Croatia*

56. Ninoslav Truhar, *Department of Mathematics, University of Osijek, Croatia*
57. Matea Ugrica, *Department of Mathematics, University of Josip Juraj Strossmayer in Osijek*
58. Anja Vrbaški, *Faculty of Mining, Geology and Petroleum Engineering*
59. Marko Vrdoljak, *University of Zagreb, Faculty of Science, Department of Mathematics*

# Programme

Opening and closing, as well as all lectures take place in (i.e., will be broadcasted to/from) the conference room **Kastrum**.

**Monday, September 14th**

| | |
|---|---|
| 09:00 - 09:05 | Opening |
| 09:05 - 09:55 | Plenary: Pileckas |
| 09:55 - 10:20 | Caggio |
| 10:20 - 10:45 | Kampschulte |
| 10:45 - 11:15 | Coffee break |
| 11:15 - 11:40 | Schwarzacher |
| 11:40 - 12:05 | Kalousek |
| 12:05 - 12:30 | Radulović |
| 12:30 - 14:00 | Lunch |
| 14:00 - 14:50 | Plenary: Benner |
| 14:50 - 15:15 | Mlinarić |
| 15:15 - 15:40 | Tomljanović |
| 15:40 - 16:10 | Coffee break |
| 16:10 - 16:35 | Ugrica |
| 16:35 - 17:00 | Šain Glibić |
| 17:00 - 17:25 | Miodragović |
| 17:25 - 17:50 | Carević |

**Tuesday, September 15th**

| | |
|---|---|
| 09:05 - 09:55 | Plenary: Veselić |
| 09:55 - 10:20 | Pašić |
| 10:20 - 10:45 | Antonić |
| 10:45 - 11:15 | Coffee break |
| 11:15 - 11:40 | Sperone |
| 11:40 - 12:05 | Kovač |
| 12:05 - 12:30 | Bašić-Šiško |
| 12:30 - 14:15 | Lunch |
| 14:15 - 14:40 | Prša |
| 14:40 - 15:05 | Galić |
| 15:05 - 15:30 | Rosenbaum |
| 15:30 - 16:00 | Coffee break |
| 16:00 - 16:50 | Plenary: Gugercin |
| 16:50 - 17:15 | Gosea |
| 17:15 - 17:40 | Drmač |
| 17:40 - 18:05 | Palitta |

**Wednesday, September 16th**

| | |
|---|---|
| 09:05 - 09:55 | Plenary: Zunino |
| 09:55 - 10:20 | Marušić-Paloka |
| 10:20 - 10:45 | Ljulj |
| 10:45 - 11:15 | Coffee break |
| 11:15 - 11:40 | Radišić |
| 11:40 - 12:05 | Bukac |
| 12:05 - 12:30 | She |
| 12:30 - 14:00 | Lunch |
| 14:00 - 14:25 | Burazin |
| 14:25 - 14:50 | Jankov |
| 14:50 - 15:15 | Tambača |
| 15:15 - 15:40 | Crnjac |
| 15:40 - 19:30 | Free time |
| 19:30 - 00:00 | Conference dinner |

**Thursday, September 17th**

| | |
|---|---|
| 09:05 - 09:55 | Plenary: Simoncini |
| 09:55 - 10:20 | Grubišić |
| 10:20 - 10:45 | Begović Kovač |
| 10:45 - 11:15 | Coffee break |
| 11:15 - 11:40 | Bosner N. |
| 11:40 - 12:05 | Ruymbeek |
| 12:05 - 12:30 | Truhar |
| 12:30 - 14:15 | Lunch |
| 14:15 - 14:40 | Astrauskas |
| 14:40 - 15:05 | Sheloput |
| 15:05 - 15:30 | Macha |
| 15:30 - 16:00 | Coffee break |
| 16:00 - 16:50 | Plenary: Galdi |
| 16:50 - 17:15 | Gravina |
| 17:15 - 17:40 | Necasova |
| 17:40 - 18:05 | Radošević |
| 18:05 - 18:30 | Češík |

**Friday, September 18th**

| | |
|---|---|
| 09:05 - 09:55 | Plenary: Heltai |
| 09:55 - 10:20 | Kunštek |
| 10:20 - 10:45 | Vrdoljak |
| 10:45 - 11:15 | Coffee break |
| 11:15 - 11:40 | Bosner T. |
| 11:40 - 12:05 | Gupta |
| 12:05 - 12:30 | Geridönmez |
| 12:30 - 12:55 | Bejenaru |
| 12:55 - 13:00 | Closing |
| 13:00 - 14:30 | Lunch |

# Invited Talks

## Balancing-based model reduction methods for nonlinear systems

Peter Benner

Computational Methods in Systems and Control Theory (CSC), Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany

benner@mpi-magdeburg.mpg.de

During the past three decades, Model Order Reduction (MOR) has been established as an important tool to overcome complexity barriers at the intersection of nearly all disciplines in the computational sciences and engineering (CSE). MOR generates surrogate models for numerical simulation that are much faster to evaluate than the original mathematical models of dynamical processes, but retain the necessary numerical accuracy for the quantities-of-interest that are necessary when using the model for control, prediction or verification purposes.

Here, we discuss MOR for nonlinear systems from a system-theoretic perspective, focusing on the approximation of the mapping from inputs (controls, design parameters) to outputs (measurements, quantities-of-interest). Typical methods in this area are either based on system balancing or rational interpolation. These methods are well established for linear systems and part of CSE and control design software packages. Recent years have seen a major effort in generalizing these methods to nonlinear systems. Here, we will in particular discuss rcent advances in balancing-based methods for nonlinear systems. The performance of the new methods is illustrated for several benchmark examples, typically arising from the spatial discretization of time-dependent nonlinear partial differential equations.

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## On the self-propelled motion of a rigid body in a viscous liquid by time-periodic driving boundary data

Giovanni P. Galdi

University of Pittsburgh, Department of Mechanical Engineering and Materials Science

galdi@pitt.edu

We consider a body, $\mathcal{B}$, moving in a Navier-Stokes liquid, $\mathcal{L}$, subject to a driving mechanism constituted by a time-periodic distribution of velocity, $\mathbf{v}_*$, at the interface body-liquid. This study is mostly motivated by the self-propulsion of a "fish", where the net motion is produced by the continuous oscillation of parts of its body. Even though modeling a fish as a rigid body and its moving parts as a boundary velocity distribution might look a bit coarse, it should also be said that, as is well known, the motion of a (deformable) shape-changing object in a liquid can mathematically be reduced – by a suitable transformation – to that of an object of fixed shape with an appropriate distri- bution of velocity at its boundary. In particular, we show that, in a suitable class of weak solutions, if the average over a period of $\mathbf{v}_*$, $\overline{\mathbf{v}}_*$ is not zero, then $\mathcal{B}$ will propel itself on condition that $\overline{\mathbf{v}}_*$ has a non-vanishing projection on a suitable "control" space. If, on the other hand, $\overline{\mathbf{v}}_* = \mathbf{0}$ (purely oscillatory case), then we show that self-propulsion can occur if and only if $\overline{\mathbf{v}}_*$ satisfies a suitable non-local condition.

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## Data-driven modeling for dynamical systems

Serkan Gugercin

Virginia Tech, Blacksburg, Virginia, USA

gugercin@vt.edu

In this talk, we will investigate various approaches to modeling dynamical systems from data. We will focus on the case in which the data result from the frequency-domain measurements (transfer function samples) of an underlying dynamical system. Barycentric form of a rational function will form the foundation of how we construct the data-driven approximants for both (parametric) linear dynamical systems and for linear dynamical systems with quadratic outputs. We will also show how these data-driven approximants can be used to estimate dispersion curves from experimental data in structural dynamics.

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## Non-matching frameworks for the simulation of coupled problems

Luca Heltai
SISSA Trieste
luca.heltai@sissa.it

Partial differential equations with interfaces, holes, cracks, or defects often require the numerical solution of coupled problems on domains with different topological dimension. In this talk, I will discuss and analyse some techniques that can be used to tackle this class of problems, using non-matching discretisations that combine Finite Element Methods, regularization techniques, weighted Sobolev spaces, and reduce order models.

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## Recent years achievements in the theory of stationary Navier-Stokes equations

Konstantin Pileckas
Institute of Applied Mathematics, Vilnius University, Vilnius, Lithuania
konstantinas.pileckas@mif.vu.lt

All results of this presentation were obtained jointly with M. Korobkov and R. Russo.

In the first part of the talk, I will present recent results on the Leray problem (1933) for the nonhomogeneous boundary value problem for the steady Navier-Stokes equations in a bounded domain with multiple connected boundary. The boundary conditions are assumed only to satisfy the necessary requirement of zero total flux. It is proved that the problem is solvable in arbitrary bounded planar or three-dimensional axially symmetric domains. The proof uses Bernoulli's law for weak solutions of the Euler equations and a generalization of the Morse-Sard theorem for functions in Sobolev spaces.

In the second part of the talk, I will discuss the existence results for the steady Navier-Stokes equations in two- and three-dimensional exterior domains with multiply connected boundaries (in the three-dimensional case under the assumption of axial symmetry) and nonhomogeneous boundary conditions. We prove that for 3D case, this problem has a solution for axially symmetric data without any restrictions on the fluxes of the boundary value. For the two-dimensional case the problem with nonhomogeneous boundary conditions admits at least one solution with finite Dirichlet integral if the total flux of the boundary value is zero.

Finally, I will discuss the properties of the arbitrary solutions with the finite Dirichlet integral of the two-dimensional exterior problem for the stationary Navier-Stokes equations. It will be shown that such solutions are bounded and uniformly converge to a constant vector at infinity. Moreover, we prove that the Leray's solution to the obstacle problem (flow past a prescribed body) is always nontrivial. By Leray's solutions we understand the ones obtained by the method of "invading domains". Denoting by $\mathbf{u}_k$ the solution to the Navier-Stokes problem on the intersection $\Omega_k$ of the domain $\Omega$ with the disk $B_k$ of radius $k$, J. Leray showed that the sequence $\{\mathbf{u}_k\}$ has uniformly bounded Dirichlet integral. Hence, it is possible to extract a subsequence which weakly converges to the solution $\mathbf{u}_L$ of the Navier-Stokes problem in the exterior domain $\Omega$. This solution was later called Leray's solution. However, if its limit at infinity is zero, this solution is trivial: $\mathbf{u}_L$ is identically zero. So, our results exclude this possibility.

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## Solving multiterm linear matrix equations: order reduction strategies

Valeria Simoncini

Università di Bologna

valeria.simoncini@unibo.it

Linear matrix equations have arisen as a natural formulation for many discretization methodologies on regular domains, such as finite differences, isogeometric analysis, spectral methods, and all-at-once time dependent forms. In all these cases, the discretization is characterized by large scale Kronecker operations, that can also be written in matrix or tensor form, with several terms. In the case of two terms, the algebraic problem reduces to the generalized Sylvester equation. The properties of the problem may be such that additional structure - such as low rank and/or banded coefficient matrices - is available.

We present a general framework for reducing the order of the problem in a way that depends linearly on the dimensions of the coefficient matrices, and is able to exploit the inherent spectral and structural properties of the given problem. The applicability on a variety of discretization settings will be illustrated.

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## Scale-free uncertainty relations and applications in spectral and control theory

Ivan Veselić

TU Dortmund

ivan.veselic@math.tu-dortmund.de

We present an uncertainty relation for spectral projectors of Schroedinger operators on bounded and unbounded domains. These have sevaral applications, among others in the spectral theory of random Schroedinger operators. Here we will present two applications which are likely to be of interest to the audience of the conference: Shifting of bands of the essential spectrum and of eigenvalues of Schroedinger operators and controllability of the heat equation.

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# Mixed dimensional partial differential equations: analysis, approximation and applications

Paolo Zunino

MOX, Department of Mathematics, Politecnico di Milano

paolo.zunino@polimi.it

Coupled partial differential equations (PDEs) defined on domains with different dimensionality are usually called mixed dimensional PDEs. We address mixed dimensional PDEs on three-dimensional (3D) and one-dimensional domains, giving rise to a 3D-1D coupled problem. Such a problem is not well investigated yet form the standpoint of mathematical analysis and numerical approximation, although it arises in applications of paramount importance such as microcirculation, flow through perforated media and the study of reinforced materials, just to make a few examples.

We address this mathematical problem within a general framework, designed to formulate and approximate coupled PDEs on manifolds with heterogeneous dimensionality using topological model reduction tools. The main difficulty consists of the ill-posedness of restriction operators (such as the trace operator) applied on manifolds with co-dimension larger than one. We will overcome this challenge by means of nonlocal restriction operators that combine standard traces with mean values of the solution on low dimensional manifolds. This new approach has the fundamental advantage of enabling the approximation of the problem using Galerkin projections on Hilbert spaces, which can not be otherwise applied because of regularity issues. Furthermore, combining the numerical error analysis with the model reduction approach, the concurrent modeling and discretization errors in the approximation of the original fully dimensional problem can be estimated.

We will discuss applications of such an approach to problems of significant impact in medicine and geophysics. In particular we will address its application to microcirculation and to the simulation of perforated subsurface reservoirs.

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# Contributed Talks

## H-measures and propagation of microlocal energy density

Nenad Antonić
University of Zagreb
nenad@math.hr

H-measures were successfully used for computing microlocal energy density in the linear wave equation by Francfort and Murat (1992), and for linear hyperbolic systems by Antonić and Lazar (2002). A much more challenging problem, while still within the linear theory, is such propagation in the linearised elasticity, where different types of waves occur (P and S waves). In the talk, some classes of symbols suitable for describing the transport of H-measures will be discussed, as well as the interplay of localisation and propagation principles in describing the propagation of H-measures.

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## Modelling of Scanning Electrochemical Microscope and the Influence of Eletrode Geommetry

Rokas Astrauskas[1], Feliksas Ivanauskas[1], Inga Morkvenaite-Vilkonciene[2], Arunas Ramanavicius[1]
Vilnius University[1], Vilnius Gediminas Technical University[2]
Rokas.Astrauskas@mif.vu.lt

Scanning electrochemical microscopy (SECM) is an advanced electrochemical method, which is based on electrochemical scanning of the surface by an ultramicroelectrode (UME). Due to the high complexity of the system, the evaluation of data collected by this electrochemical method is a challenging task, which can be simplified by mathematical modeling of processes that are influencing the electric current, measured by SECM.

In the first part of the talk, the mathematical model of SECM, acting in redox competition mode, is presented. The model is governed by the system of 8 nonlinear reaction-diffusion equations. The system is solved by alternating-direction finite difference method and Runge-Kutta method. The numerical solution is compared with results of physical experiment, and by fitting modelling data to the experiment, reaction coefficients and diffusion parameters are calculated.

Variations of UME geometry can decrease accuracy of the measurement, and then correct analysis of the SECM data becomes almost impossible. In the second part of the talk, we study the precision of measurements with three different and most frequent types of defected UMEs: (i) recessed-UME, (ii) outwarded-UME, (iii) cone-UME. These electrodes are mathematically modelled by diffusion equations in various non-standard (non-rectangular) domains. Equations are solved numerically and the results compared with that obtained with not defected plane-UME in order to calculate the difference between standard geometry and non-standard geometries. The correctness of the model is tested by comparing computations for recessed-UME model with data of the physical experiment.

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## Local Solvability for Micropolar Real Gas Flow Model

Angela Bašić-Šiško[1], Ivan Dražić[2]

University of Rijeka, Faculty of Engineering

abasicsisko@riteh.hr[1], idrazic@riteh.hr[2]

We analyze the micropolar, viscous, polytropic, and heat-conducting real gas, whereby we assume the generalized form of pressure function in the sense that pressure is the affine function of temperature and power function of mass density. Using the stated thermodynamical and constitutive assumptions, we derive a one-dimensional model based on balance laws.

Using Faedo-Galerkin method we construct a sequence of approximate solutions to our problem and prove that the generalized solution exists locally in time. We also solve the problem numerically using the obtained approximate solutions.

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## Partially projective algorithm for the split feasibility problem with visualization of the solution set

Andreea Bejenaru[1], Mihai Postolache[2]

University Politehnica of Bucharest, Romania[1,2], Center for General Education, China Medical University, Taiwan and Gh. Mihoc-C. Iacob Institute of Mathematical Statistics and Applied Mathematics, Romanian Academy, Bucharest, Romania[2]

bejenaru.andreea@yahooo.com[1], mihai@mathem.pub.ro[2]

Our study introduces a new three-step algorithm to solve the split feasibility problem. The main advantage is that one of the projective operators interferes only in the final step, resulting less computations at each iteration. An example is provided to support the theoretical approach. The numerical simulation reveals that the newly introduced procedure has increased performance compared to other existing methods, including the classic CQ algorithm. An interesting outcome of the numerical modeling is an approximate visual image of the solution set.

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## Hybrid CUR-type decomposition of tensors in the Tucker format

Erna Begović Kovač
University of Zagreb
ebegovic@fkit.hr

In recent years attention is given to low-rank approximations obtained by interpolatory factorizations. These approximations, although suboptimal, keep the properties like sparsity and non-negativity. Thus, they are more suitable in some applications where the fibers (columns/rows/tubes) should keep their original meaning. The best known example of the interpolatory factorizations is CUR factorization.

In this talk we introduce a hybrid approach to the CUR-type decomposition of tensors in the Tucker format. The main idea of the hybrid algorithm is to write a tensor $\mathcal{A}$ as a product of a core tensor $\mathcal{S}$, a matrix $C$ obtained by extracting mode-$k$ fibers of $\mathcal{A}$, and matrices $U_j$, $j = 1, \ldots, k-1, k+1, \ldots, d$, chosen to minimize the approximation error $\mathcal{E}$,

$$\mathcal{A} = \mathcal{S} \times_1 U_1 \times_2 \cdots \times_{m-1} U_{m-1} \times_m C_m \times_{m+1} U_{m+1} \cdots \times_d U_d + \mathcal{E}.$$

The approximation error obtained this way is smaller than the one coming from the standard tensor CUR-type method. This difference gets more important as the tensor dimension increases. We give the error bound for the new method and compare it to the error resulting from the standard CUR approach.

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## Joint Approximate Diagonalization of Several Matrices by an Optimization Algorithm on a Matrix Manifold

Nela Bosner
Faculty of Science, Department of Mathematics, University of Zagreb
nela@math.hr

Rank revealing CP decomposition is a tensor factorization which is heavily exploited in applications. In the symmetrical case, by projecting a tensor along multiple random vectors the tensor factorization reduces to the problem of joint diagonalization of several symmetric matrices. In the presence of noise it is not possible to obtain the exact joint diagonalization, and the problem is transformed to the optimization problem of finding nearly diagonal form. Existing algorithms for joint diagonalization use standard optimization algorithms and if necessary impose some additional constraint on the transformation matrix in order to secure its nonsingularity. Since the joint diagonalization algorithm represents a core of the tensor factorization, our goal was to enhance existing algorithms by combining diagonalization of the random linear combination of the matrices with the optimization algorithm (like Newton method or Conjugate Gradient method) on an appropriate matrix manifold, thus avoiding additional constraints. We are going to provide derivation and analysis of the algorithm in both the differential geometry and numerical analysis framework, and produce algorithm implementation.

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## Quasi-collocation method based on CCC–Schoenberg operators

Tina Bosner
Department of Mathematics, Faculty of Science, University of Zagreb
tinab@math.hr

We propose a quasi-collocation method for solving second order boundary value problems $L_2 y = f$, in which the differential operator $L_2$ can be represented in the product formulation, aiming mostly on singular and singularly perturbed boundary value problems. If we seek an approximating Canonical Complete Chebyshev spline $s$ by some collocation method, we then actually demand that $L_2 s$ interpolate the function $f$. On the other hand, in our method we require that $L_2 s$ is equal to the approximation of $f$ by the Schoenberg operator, associated with the second reduced system. By doing that, we can avoid solving any linear system. We also show the quadratic convergence of such an approximation, and suggest two algorithms for its calculation.

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## Partitioned numerical methods for fluid-structure interaction problems

Martina Bukač
University of Notre Dame
mbukac@nd.edu

Fluid-structure interaction (FSI) problems arise in many applications, such as geomechanics, aerodynamics, and blood flow dynamics (hemodynamics). Computational algorithms for FSI problems can be classified as monolithic or partitioned. In the monolithic approach, the fluid and structure equations are strongly coupled, but the resulting linear system is large and ill-conditioned. Alternatively, using the partitioned approach, the fluid problem is solved separately from the structure problem, resulting in two smaller and better-conditioned linear systems. However, stability issues often arise as a result of the coupling at the interface unless the design and implementation of a partitioned scheme is carefully developed.

We present a partitioned, loosely-coupled scheme for the interaction between an incompressible, viscous fluid and a thin structure called a BOundary Update using Resolvent (BOUR) partitioned method. We show that the method is second-order accurate in time and unconditionally stable. The method is algorithmically similar to the sequential Backward Euler - Forward Euler implementation of the midpoint

quadrature rule. (i) The structure and fluid sub-problems are first solved using a Backward Euler scheme, (ii) the velocities of fluid and structure are updated on the boundary via a second-order consistent resolvent operator, and then (iii) the structure and fluid sub-problems are solved again, using a Forward Euler scheme.

Using similar approach based on the sequential Backward Euler - Forward Euler implementation of the one-legged $\theta$ scheme, we also present a strongly-coupled partitioned method for the interaction between the fluid and a thick structure. The proposed method is stable and second-order accurate provided $\theta = \frac{1}{2}$. Performance of both methods is demonstrated using numerical examples.

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## Optimal design of transversely loaded plate

Krešimir Burazin[1], Ivana Crnjac[2], Jelena Jankov[3]
University of Osijek[1,2,3]
kburazin@mathos.hr[1], icrnjac@mathos.hr[2], jjankov@mathos.hr[3]

The development of general, non-periodic homogenization theory for second order elliptic partial differential equations was initiated by Spagnolo through the concept of G-convergence (1968), and further generalized by Tartar (1975) and Murat and Tartar (1978) under the name H-convergence. Some aspects for higher order elliptic problems were considered by Zhikov, Kozlov, Oleinik and Ngoan (1979) and Antonić and Balenović (1999, 2000). The theory proved to be useful for treatment of optimal design problems, where the goal is to find the best arrangement of given materials within the body, which optimizes its properties with respect to some optimality criteria. This optimality of the distribution is usually expressed as a minimization (maximization) of an integral functional. Murat and Tartar (1985) showed that the homogenization method, where a mixture of original materials on micro scale is used as a generalized design, gives a proper relaxation of the original problem. This approach also enabled the development of one of the most popular numerical methods for finding approximate solution of the optimal design problems, namely, the optimality criteria method, an iterative method based on necessary conditions for optimality of the relaxed formulation. The whole approach proved to be more fruitful for the stationary diffusion equation, where the set of generalized designs is explicitly known, while in linearized elasticity this approach is mostly limited to the compliance optimization problems, and relies on the information about bounds on the effective energy of a composite material, known as Hashin-Shtrikman bounds.

A recent development (Burazin, Jankov, Vrdoljak, 2018) of the general homogenization theory for the Kirchhoff-Love elastic plate equation, which models pure bending of a thin solid symmetric plate under a transverse load, enables us to derive some novel results regarding optimal design for elastic plates. Firstly, we prove that the relaxation by the homogenization method is a proper relaxation of the original problem. Then we derive a necessary condition of optimality for the relaxed problem. Finally, we explicitly calculate the corresponding Hashin-Shtrikman bound, and develop a new variant of the optimality criteria method for elastic plates.

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## Dimension reduction in fluid mechanics

Matteo Caggio
Department of Mathematics, Faculty of Science, University of Zagreb
matteo.caggio@math.hr

We consider the compressible Navier-Stokes system describing the motion of a viscous fluid confined to a straight layer $\Omega = (0, \delta) \times \mathbb{R}^2$. We show that the weak solutions in the 3D domain converge to the solution of the 2D incompressible Navier-Stokes equations (Euler equations) when the Mach number tends to zero as well as $\delta \to 0$ (and the viscosity goes to zero), [1]. An application of this analysis to the compressible Euler equation in the framework of measure-valued solutions and an extension to heat conducting fluids is shortly discussed, [2,3].

### References

[1] Low Mach number limit on thin domains (M. Caggio, D. Donatelli, Š. Nečasová, Y. Sun), Nonlinearity 33 (2020) 840-863.

[2] Low Mach and thin domain limit for the compressible Euler system (M. Caggio, B. Ducomet, Š. Nečasová, T. Tang), arXiv:2001.09162.

[3] Low Mach, stratification and thin domain limit for the compressible Navier-Stokes-Fourier system (M. Caggio, D. Donatelli, Š. Nečasová, M. Pokorný), in preparation.

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## Determination of the regularization parameter for the distorted Born iterative method

Anita Carević
University of Split
carevica@fesb.hr

The distorted Born iterative (DBI) method can be used to solve the problem of ultrasound tomography. However, choosing appropriate regularization parameter $\lambda$ which provides an optimal balance between overregularized and underregularized solution is essential for this method to converge. We present a new adaptive algorithm for choosing $\lambda$. It is based on minimizing two inversely proportional components: signal loss and noise error. It starts with an overestimation of the noise in the measured data which is appropriately adjusted within iterations of the DBI method using the discrepancy between measured and calculated data. Through numerical simulations of ultrasound tomography we compare our algorithm to standard algorithms for parameter search such as the L-curve and Generalized cross-validation (GCV).

This is a joint work with professors Ivan Slapničar (University of Split), Jesse Barlow and Mohamed Almekkawy (Pennsylvania State University)

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## Hashin-Shtrikman bounds and optimal design problems in linearized elasticity

Krešimir Burazin[1], Ivana Crnjac[2], Marko Vrdoljak[3]
University of Osijek[1,2], University of Zagreb[3]
kburazin@mathos.hr[1], icrnjac@mathos.hr[2], marko@math.hr[3]

In optimal design problems, the goal is to find the arrangement of given materials within the body, which optimizes its properties with respect to some optimality criteria. Optimality of the distribution is usually expressed as a minimization (maximization) of an integral functional depending on rerrangement of materials that constitute the domain and solution of a partial differential equation modelling the involved physics.

We consider an elastic domain whose mechanical behaviour under some force load is described with the linearized elasticity system of PDEs, and restrict ourselves to domains filled with two isotropic elastic phases. Since the classical solution of optimal design problems usually does not exist, we use relaxation by the homogenization method in order to get a proper relaxation of the original problem. Unfortunately, G-closure is not known in linearized elasticity, not even for mixtures of two isotropic phases. However, in some special cases, Hashin-Shtrikman bounds on the G-closure set prove useful. More precisely, in the case of minimizing a functional that is equal to the total elastic energy of the system, the minimization can be performed on a smaller subset made of sequential laminates, which is explicitly known. Furthermore, for the compliance functional the necessary conditions of optimality are easily derived, which enables a development of optimality criteria method for the numerical solution. Since the lower Hashin-Shtrikman bound arises in the necessary conditions of optimality for this problem, it's explicit calculation is needed. We give it both in two and three space dimensions and present a number of examples of two-dimensional compliance minimization problems.

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## Convex hull properties for parabolic systems of partial differential equations

Antonín Češík
Charles University
cesik@karlin.mff.cuni.cz

We examine a generalisation of the well-known maximum principles for (scalar-valued) solutions of PDE to the case of systems of PDE (i.e. with $\mathbb{R}^N$-valued solutions) – the *convex hull property*. This property says that the values of a function lie within the convex hull of its boundary values. The main novelty of this work is a proof of the convex hull property for vector-valued parabolic $p$-Laplace equation, with the possibility of including lower-order terms.

It is curious that well-behaved scalar non-linearities in the system do not spoil the convex hull property, even though not much can be allowed with regards to the coupling of the system's coefficients. We will see that for linear elliptic systems, the condition is that the system consists of scalar equations for each of the components (possibly up to an orthogonal change of variables), and provide a counterexample otherwise. Similarly for the case of linear parabolic equations, where the condition is even more rigid.

This is a joint work with L. Diening and S. Schwarzacher.

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## Numerical linear algebra for the Koopman and the dynamic mode decompositions

Zlatko Drmač

Department of Mathematics, Faculty of Science, University of Zagreb, Croatia

drmac@math.hr

The Dynamic Mode Decomposition (DMD, introduced by P. Schmid) has become a tool of trade in computational data driven analysis of complex dynamical systems, e.g. fluid flows, where it can be used to decompose the flow field into component fluid structures, called DMD modes, that describe the evolution of the flow. The DMD is deeply connected with the Koopman spectral analysis of nonlinear dynamical systems, and it can be considered as a computational device in the Koopman analysis framework. Its exceptional performance motivated developments of several modifications that make the DMD an attractive method for analysis and model reduction of nonlinear systems in data driven settings.

In this talk, we will present our recent results on the numerical aspects of the DMD/Koopman analysis. We show how the state of the art numerical linear algebra can be deployed to improve the numerical performances in the cases that are usually considered notoriously ill-conditioned. Further, we show how even in the data driven setting, we can work with residual bounds, which allows error estimates for the computed modes.

Our presentation is based on the recent papers

1. Z. Drmač, I. Mezić, R. Mohr: *On least squares problems with certain Vandermonde–Khatri–Rao structure with applications to DMD.* SIAM Journal on Scientific Computing, (2020) in print. (arXiv:1811.12562)

2. Z. Drmač: *Dynamic Mode Decomposition – A Numerical Linear Algebra Perspective*, The Koopman Operator in System and Control (Lecture Notes in Control and Information Sciences 484), A. Mauroy, I. Mezić and Y. Susuki (Eds)", Springer 2020, pp. 161-194

3. I. Mezić, Z. Drmač, N. Črnjarić-Žic, S. Maćešić, M. Fonoberova, R. Mohr, A. M. Avila, I. Manojlović, A. Andrejčuk: *A Koopman operator based prediction algorithm and its application to COVID-19 pandemic*, Nature Communications, in revision 2020.

4. Z. Drmač, I. Mezić, R. Mohr: *Data driven Koopman spectral analysis in Vandermonde-Cauchy form via the DFT: numerical method and theoretical insights*. SIAM Journal on Scientific Computing, 41 (5) (2019), pp. A3118–A3151. (arXiv:1808.09557)

5. Z. Drmač, I. Mezić, R. Mohr: *Data driven modal decompositions: analysis and enhancements*, SIAM Journal on Scientific Computing, 40 (4) (2018), pp. A2253–A2285. (arXiv:1708.02685)

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## A model in one-dimensional thermoelasticity

Tomasz Cieślak[1], Marija Galić[2], Boris Muha[3]

Institute of Mathematics, Polish Academy of Sciences[1], Department of Mathematics, Faculty of Science, University of Zagreb[2,3]

cieslak@impan.pl[1], marijag5@math.hr[2], borism@math.hr[3]

We study a one-dimensional nonlinear hyperbolic-parabolic initial boundary value problem occurring in the theory of thermoelasticity. We prove existence and uniqueness of the local-in-time strong solution as well as existence of some global-in-time weak measure valued solutions. To this end we introduce an auxiliary problem with artificial viscosity and prove its global-in-time well-posedness. Next, we show that solutions of the auxiliary problem converge, at some short time interval to the strong solution, and to our measure valued solution for an arbitrary time.

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## A new regression based approach to solve a heat transfer problem

Bengisen Pekmen Geridönmez

TED University, Department of Mathematics

bengisenpekmen@gmail.com

In this study, a novel statistical approach is proposed to solve a free convection problem. Multivariate adaptive regression splines (Mars) is utilized to determine model functions for unknowns of dimensionless governing equations formulated by stream function and vorticity, without the need of an iterative solver. Once the truth data is obtained as a result of the numerical solution of the dimensionless, time dependent equations by polyharmonic radial basis function based pseudo spectral (Rbf-Ps) method, model functions for stream function, temperature and vorticity are produced by Mars. In the original problem, these model functions are used anymore. This enables one to be independent from a solver. Results are also validated with a benchmark study, and well agreement is obtained. Model function built is also tested against the results of the original method.

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## A hybrid model reduction approach for linear dynamical systems - connections to data-driven methods

Ion Victor Gosea[1], Serkan Gugercin[2], Christopher Beattie[3]
Max Planck Institute, Magdeburg, Germany[1], Virginia Tech University, Blacksburg, Virginia, USA[2,3].
gosea@mpi-magdeburg.mpg.de[1], gugercin@vt.edu[2], beattie@vt.edu[3]

In this contribution, we propose a systems theoretical model reduction approach inspired by the AAA algorithm originally introduced in [1]. It is a hybrid procedure since it uses the adaptive and iterative properties characterizing AAA. At the same time, the sampling-based discrete error measures are replaced with continuous ones, e.g., the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms. The algorithm for computing the reduced model (low-order rational approximant) will follow the spirit and specifications of the original AAA algorithm. More precisely, it interpolates where the error is maximal (greedy selection step) and uses the remaining degrees of freedom to minimize a least squares measure. These steps are replaced with steps that use systems-theoretic measures.

For the newly proposed method the greedy selection step is modified as follows: choose the next interpolation point that corresponds to the maximum deviation in the $\mathcal{H}_\infty$ norm. Then, select the remaining variables in order to minimize the $\mathcal{H}_2$ norm between the original and the reduced-order system. Note that the latter computation involves deriving factors of infinite Gramians, as in the case of the balanced truncation. Moreover, the Loewner matrix that appears in the original AAA algorithm will be replaced by a hybrid matrix, i.e., a "GramLoew" matrix that can be factorized in terms of a Gramian factor and a Loewner matrix factor.

Additionally, we discuss data-driven extensions of the proposed method that use only measurements of the transfer function evaluated at appropriately chosen sampling points. Finally, we present several numerical examples to illustrate the methods under consideration.

[1] Y. Nakatsukasa, O. Séte, and L. N. Trefethen, *The AAA algorithm for rational approximation*, SIAM Journal on Scientific Computing, 40(3), A1494–A1522, 2018.

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## Criteria for particle rebound in the absence of collisions

Giovanni Gravina[1], Sebastian Schwarzacher[2], Ondřej Souček[3], Karel Tůma[4]
Charles University[1,2,3,4]
gravina@karlin.mff.cuni.cz[1], schwarz@karlin.mff.cuni.cz[2], soucek@karel.troja.mff.cuni.cz[3], ktuma@karlin.mff.cuni.cz[4]

In this talk, we will discuss some analytical conditions for the rebound of an elastic body moving towards a wall in a viscous incompressible fluid. We will focus on the delicate case of no-slip boundary conditions, where fascinating phenomena can be observed due to the fact that particle-wall collisions cannot occur in finite time.

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## Mesh adaptive solution method for Lyapunov equations

Luka Grubišić
Department of Mathematics, Faculty of Science, University of Zagreb
luka.grubisic@math.hr

In this talk we present an adaptive solution method for finite element discretization of the Lyapunov equation. Our method is based on the extended Arnoldi algorithm of Simoncini and the auxiliary subspace error estimation technique for finite element discretization methods. We compare our solution to alternative solution methods for the Lyapunov equation such as QTT MKE method of Osledets.

This is a joint work with Daniel Kressner.

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## A Chebyshev collocation method for the numerical solution of space-time variable order fractional wave equation with damping

Rupali Gupta[1], Sushil Kumar[2]
Sardar Vallabhbhai National Institute of Technology Surat, Surat
rupaligupta971@gmail.com [1], skumar.iitr@gmail.com[2]

Fractional calculus has been used by many researchers due to its wide popularity and importance in various areas of science and engineering. The mathematical model with fractional order derivative shows more real phenomena compare with integer order because of its nonlocal characteristics. Bioengineering, viscoelasticity, electronics, robotics, control theory, signal processing and many other fields where the role of fractional order can be seen.

Variable order fractional calculus is the advancement of constant order fractional calculus. From last few years a significant attention has been gained by variable order fractional derivative where the order of fractional derivative can depend and vary with space, time or other parameters.The order of derivative takes a lot more importance in the modeling of physical phenomena as most of the mathematical model described in terms of differential operators. For example, in the reaction kinetics of protein where, the order indicates the nature of relaxation mechanism. On the similar note, in the modeling of various kind of damping, the characteristic of damping may depend upon current positing of damper and other parameters like temperature. In such cases the order of derivative also affected by the internal and external variables and this is the case where the use of variable order fractional calculus can be helpful. In spite of its larger importance in modeling of real life problem, finding the analytical solution is still a difficult task. Hence computational and approximation methods are proposed to achieve the approximate solution.

In this paper, we discuss the following space-time variable order fractional diffusion wave equation with damping

$$\begin{aligned}
&\tfrac{\partial^{1+\alpha(x,t)}v}{\partial t^{1+\alpha(x,t)}} + \tau \tfrac{\partial^{\alpha(x,t)}v}{\partial t^{\alpha(x,t)}} = \tfrac{\partial^{\beta(x,t)}v}{\partial x^{\beta(x,t)}} + g\left(x,t\right), \quad 0 < x < L,\ 0 \le t \le T\\
&v\left(x,0\right) = V_1\left(x\right),\ \left.\tfrac{\partial v}{\partial t}\right|_{t=0} = V_2\left(x\right), \quad 0 \le x \le L,\\
&v\left(0,t\right) = v\left(L,t\right) = 0, \quad 0 \le t \le T,
\end{aligned} \tag{1}$$

where $\tau > 0$. The fractional derivative is defined in Caputo sense and $V_1, V_2$ both are real valued well behaved functions. Due to the term $\tau \frac{\partial^{\alpha(x,t)} v}{\partial t^{\alpha(x,t)}}$, equation (1) is different from the classical wave equation in which $\tau = 0$.

We use Chebyshev collocation method to convert the considered problem into set of linear algebraic equations, which can be simply solved to get the numerical solution. The advantage of using Chebyshev polynomial as orthogonal basis function is that in the computational process the truncation error approaches zero faster than any other orthogonal polynomials. We solved few examples to show that the proposed method is reliable and most efficient. Convergence of the method is also presented We also observed that by using variable order derivatives, a lesser number of basis functions are required to get the more accurate solution.

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## Small-amplitude homogenization of elastic plate equation

Nenad Antonić[1], Krešimir Burazin[2], Jelena Jankov[3]

University of Zagreb[1], University of Osijek[2], University of Osijek[3]

nenad@math.hr[1], kburazin@mathos.hr[2], jjankov@mathos.hr[3]

General, non-periodic homogenization theory is well developed for second order elliptic partial differential equations, where the key role plays the notion of H-convergence. It was introduced by Spagnolo through the concept of G-convergence (1968) for the symmetric case, and further generalized by Tartar (1975) and Murat and Tartar (1978) for non-symmetric coefficients under the name H-convergence. Some aspects for higher order elliptic problems were also considered by Zhikov, Kozlov, Oleinik and Ngoan (1979).

The small-amplitude homogenization consists in taking a sequence of coefficients, with difference proportional to a small parameter, and then computing the first correction in the homogenization limit. We apply this procedure to the Kirchhoff-Love model for pure bending of a thin solid symmetric plate under a transverse load. Firstly, an appropriate H-convergence setting for small perturbations has to be established. Then, we explicitly calculate the first correction in the small-amplitude homogenization limit of a sequence of periodic tensors describing material properties of the elastic plate, and finally the general non-periodic case is treated by using H-measures. We expect that these results will pave the way to applications in optimal design and inverse problems for elastic plates.

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## Existence of weak solutions to a diffuse interface model for magnetic fluids

Martin Kalousek
Institute of Mathematics of the Czech Academy of Sciences
kalousek@math.cas.cz

The talk is devoted to the existence analysis of a system of partial differential equations modeling a diffuse interface flow of two Newtonian incompressible magnetic fluids. The system consists of the incompressible Navier-Stokes equations coupled with an evolutionary equation for the magnetization vector and the Cahn-Hilliard equations. Global in time existence of weak solutions to the system will be discussed. Presented results are based on the joint work with S. Mitra and A. Schlömerkemper.

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## Minimizing movements for problems involving inertia and a variational approach to fluid-structure interactions

Barbora Benešová, Malte Kampschulte, Sebastian Schwarzacher
Department of Mathematical Analysis, Faculty of Mathematics and Physics, Charles University Prague
(benesova|kampschulte|schwarz)@karlin.mff.cuni.cz

When dealing with possibly highly nonlinear, quasistationary (i.e. inertialess) problems in continuum mechanics, de Giorgi's method of minimizing movements has long been a staple for existence proofs. However by its very nature, it has always been restricted to purely dissipative, gradient flow-type systems. In this talk we present a new two-scale method which allows us to add inertial, conservative effects by using the minimizing movements method as a stepping stone to solve an approximative time-delayed problem. Using a flow-map approach, this method is not only able to cope with problems in Lagrangian, but also with those in Eulerian and even mixed formulations. We illustrate this method by deriving existence of weak solutions to a fully nonlinear, fluid-structure interaction problem between a viscoelastic bulk solid, susceptible to large deformations and a fluid obeying the incompressible Navier-Stokes equation. For this we also introduce the proper quasistationary variational formulation.

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## Trilinear embedding for divergence-form operators


Andrea Carbonaro[1], Oliver Dragičević[2], Vjekoslav Kovač[3], Kristina Ana Škreb[4]

Università degli Studi di Genova, Dipartimento di Matematica[1], University of Ljubljana, Faculty of Mathematics and Physics[2], University of Zagreb, Faculty of Science[3], University of Zagreb, Faculty of Civil Engineering[4]

carbonaro@dima.unige.it[1], oliver.dragicevic@fmf.uni-lj.si[2], vjekovac@math.hr[3], kskreb@grad.hr[4]



We give sufficient conditions that guarantee an $\mathrm{L}^p(\Omega) \times \mathrm{L}^q(\Omega) \times \mathrm{L}^r(\Omega) \to \mathrm{L}^1(\Omega \times (0,\infty))$ embedding for a triple of divergence-form elliptic operators with complex coefficients, and with Dirichlet, Neumann, or mixed boundary conditions on $\Omega$. Conditions on the exponents and coefficient matrices are expressed in terms of the so-called *p-ellipticity*, introduced previously by the first two authors. The proof relies on a particular Bellman function, found previously by the last two authors. We also present applications of the main result to paraproducts and conical square functions associated with the corresponding operator semigroups.


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## Second order shape derivatives - numerical applications for optimal design problems


Petar Kunštek[1], Marko Vrdoljak[2]

University of Zagreb[1], University of Zagreb[2]

petar@math.hr[1], marko@math.hr[2]



We consider optimal design problems for stationary diffusion in the case of two isotropic phases. Goal is to find an optimal distribution of the phases which maximizes the energy functional. Assuming that the interface between phases is regular, one deals with transmission problems for which first and second order shape derivatives are derived. Shape derivatives can be written as a domain integral or as an integral over interface. The domain expression or distributed shape derivative seems more appropriate for numerical implementation since boundary representations consist of jumps of a functions over interface.

Descent methods based on distributed first and second order shape derivatives are implemented and tested in classes of problems for which classical solutions exist and can be explicitly calculated from optimality conditions. We have observed a stable convergence of both descent methods with second order method converging in half as many steps.


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## Nonlinear Naghdi type shell model

Matko Ljulj[1], Josip Tambača[2]

University of Zagreb[1,2]

mljulj@math.hr[1], tambaca@math.hr[2]

We propose a new nonlinear shell model of Naghdi type, well defined for shells whose middle surface is parameterised by a Lipschitz function. Behaviour of the shell is described by deformation of the middle surface and the matrix function with values in rotations which describes the rotation of the shell cross–section. The model is formulated as the minimization problem for the total energy functional that includes flexural, membrane, shear and drill energies.

We describe its properties: restrictions to a particular subset of admissible functions gives as a model of flexural or Koiter type, it is frame indifferent, its linearization gives a linear model of Naghdi type, its differential formulation implies that it is a Cosserat model with one director... After that we will analyse its asymptotic behaviour as the thickness of the shell tends to zero, in various regimes depending on the elastic properties of the material. We compare limit models (obtained by using Gamma-convergence) with rigorously derived nonlinear models obtained from 3d elasticity.

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## Compressible fluid inside of moving body

Václav Mácha

Institute of Mathematics of the Czech Academy of Sciences

macha@math.cas.cz

The presence of a compressible fluid inside a body has a tremendous effect on the dynamics of the whole system. During this talk, we present proofs of this issue for two particular systems – a freely moving body and a pendulum, both filled by a compressible fluid.

The talk is based on results obtained in collaboration with G.P.Galdi, Š. Nečasová and B. She

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# Mathematical model of heat transfer through pipe

Matko Ljulj[1], Eduard Marušić-Paloka[2], Igor Pažanin[3], Josip Tambača[4]
Departmant of Mathematics, University of Zagreb
emarusic@math.hr

The standard model for heat transfer between the fluid in the pipe and the exterior medium neglects the effects of the pipe's wall. Our goal is to prove that they are not always negligible. Comparing the ratio between diffusivities of the fluid and the wall (denoted by $\varepsilon^q$) with the wall's thickness (denoted by $\varepsilon$), using rigorous asymptotic analysis, we find five different models for effective description of the heat exchange. In all cases the effective equation remains the same heat conduction equation. The difference lies in the effective boundary condition (EBC) describing the heat exchange through the pipe's wall $S$.

- For $q < -1$ the thermal conductivity of the pipe's wall is so large that it's temperature is constant. Furthermore, the EBC is the non-local Robin :

$$\boxed{\theta = const. \quad , \quad \frac{\partial \overline{\theta}}{\partial \mathbf{n}} = Nu\left(\overline{G} - \overline{\theta}\right) \quad \text{on} \quad S \,,}$$

  where $\overline{\theta}$ and $\overline{G}$ are the average fluid and exterior temperatures, respectively.

- For $q = -1$ the thermal conductivity of the pipe's wall is large enough that longitudinal propagation at the boundary is important. The two processes are of the same order and we get the coupled boundary value problem consisting of the heat conduction equation in the fluid and the curvilinear version of the heat equation (with Laplace-Beltrami operator) in pipe's wall $S$:

$$\boxed{\frac{\partial \theta}{\partial \mathbf{n}} - \kappa \Delta_S \theta = Nu(\, G - \theta \,) \quad \text{on} \quad S \;.}$$

- For $-1 < q < 1$ no influence of the pipe's wall conductivity is present in the limit model (classical engineering case):

$$\boxed{\frac{\partial \theta}{\partial \mathbf{n}} = Nu(\, G - \theta \,) \quad \text{on} \quad S \;.}$$

- In case $q = 1$ the thermal conductivity is small enough that it reduces the effective Nusselt number in EBC from $Nu$ to $\frac{\kappa\, Nu}{\kappa + Nu}$ . The EBC becomes

$$\boxed{\frac{\partial \theta}{\partial \mathbf{n}} = \frac{\kappa\, Nu}{\kappa + Nu}\,(G - \theta) \quad \text{on} \quad S.}$$

- The last case is when $q > 1$. In that case the wall of the pipe conducts the temperature so badly that it effectively behaves as an insulator:

$$\boxed{\frac{\partial \theta}{\partial \mathbf{n}} = 0 \quad \text{on} \quad S.}$$

## Frequency isolation for the hyperbolic quadratic eigenvalue problem

Julio Moro[1], Frenando de Terán[2], Ninoslav Truhar[3], Suzana Miodragović[4]

J. J. Strossmayer University of Osijek[3,4] ,Universidad Carlos III de Madrid[1,2]

jmoro@math.uc3m.es[1], fteran@math.uc3m.es[2], ntruhar@mathos.hr[4], ssusic@mathos.hr[4]

The solution of the forced system undergo large oscillations whenever some eigenvalue of the corresponding quadratic eigenvalue problem $(\lambda^2 M + \lambda C + K)\, x = 0$, $0 \neq x \in \mathbb{C}^n$, is close to the frequency of the external force. One way to avoid resonance is to modify matrices $M$, $C$ and $K$ in such a way that the new system has no eigenvalues close to these frequencies. This frequency isolation problem is considered for the hyperbolic QEP.

## Optimization-based parametric model order reduction via Wilson-type $\mathcal{H}_2 \otimes \mathcal{L}_2$ conditions

Manuela Hund, Tim Mitchell, Petar Mlinarić, Jens Saak

Max Planck Institute for Dynamics of Complex Technical Systems

hund@mpi-magdeburg.mpg.de, mitchell@mpi-magdeburg.mpg.de, mlinaric@mpi-magdeburg.mpg.de, saak@mpi-magdeburg.mpg.de

For $\mathcal{H}_2$-optimal model order reduction of (non-parametric) linear time-invariant systems, there are a number of proposed methods in the literature, some based on the Wilson first-order necessary optimality conditions. We extend these conditions to parametric systems and propose an optimization-based method for computing locally $\mathcal{H}_2 \otimes \mathcal{L}_2$-optimal reduced-order models that are uniformly asymptotically stable. We present the suitability of our new approach on a few benchmark models.

# A weak-strong uniqueness result for 3D incompressible/compressible fluid-rigid body interaction problem

Šárka Nečasová

Institute of Mathematics, Academy of Sciences of the Czech Republic

matus@math.cas.cz

We study a 3D nonlinear moving boundary fluid-structure interaction problem describing the interaction of the fluid flow with a rigid body. The fluid flow is governed by 3D incompressible Euler equations /compressible Navier-Stokes equations, while the motion of the rigid body is described by a system of ordinary differential equations called Euler equations for the rigid body. The equations are fully coupled via dynamical and kinematic coupling conditions. We consider two different kinds of kinematic coupling conditions: no-slip (for compressible case) and slip (for incompressible case). First, we show the existence of a measure-valued solution of 3D incompressible Euler equations. Then we prove a generalization of the well-known weak-strong uniqueness result for the Euler equations coupled with the rigid body [1]. Moreover, we show also weak-strong uniqueness for the compressible fluid-rigid body system [2]. In the both cases we show that the strong solution, which is known to exist under certain smallness assumptions, is unique in the class of measure-valued / weak solutions to the problem. The proof relies on a correct definition of the relative energy, to use this tool we then have to introduce a change of coordinates to transform the strong solution to the domain of the measure-valued/ weak solution in order to use it as a test function in the relative energy inequality. Estimating all arising terms we prove that the measure-valued/ weak solution has to coincide with the transformed strong solution and finally that the transformation has to be in fact an identity, see [1,2].

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## Fast, matrix-free matrix vector product with the Löwner matrix

Davide Palitta[1], Sanda Lefteriu[2]

Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany[1], Ecole des Mines de Douai, Douai, France[2]

palitta@mpi-magdeburg.mpg.de[1], sanda.lefteriu@imt-lille-douai.fr[2]

The Löwner framework is one of the most successful data-driven model order reduction techniques. Given $k$ right interpolation data and $h$ left interpolation data, the standard layout of this approach is composed of two stages. First, the $kh \times kh$ Löwner matrix $\mathbb{L}$ and shifted Löwner matrix $\mathbb{L}_s$ are constructed. Then, an SVD of $\mathbb{L}_s - \gamma\mathbb{L}$, $\gamma \in \mathbb{C}$ belonging to one of the data sets, provides the projection matrices used to compute the sought reduced model. These two steps become numerically challenging for large $k$ and $h$ in terms of both computational time and storage demand. We show how the structure of $\mathbb{L}$ and $\mathbb{L}_s$ can be exploited to reduce the cost of performing $(\mathbb{L}_s - \gamma\mathbb{L})x$ while avoiding the explicit allocation of $\mathbb{L}$ and $\mathbb{L}_s$.

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## Positive relative oscillation in some inhomogeneous nonlinear Schrödinger equations

Mervan Pašić[1], Satoshi Tanaka[2]

FER, University of Zagreb, Croatia[1], Tohoku University, Miyagi, Japan[2]

mervan.pasic@fer.hr[1], satoshi.tanaka.d4@tohoku.ac.jp[2]

For a class of inhomogeneous nonlinear Schrödinger equations:

$$u'' + (\mu - V(x))u = g(x)u^3, \qquad \text{(NLSE-exact)}$$

where the potential difference $\mu - V(x)$ and the nonlinear potential $g(x)$ are explicitly given and generated by a function $\rho(x) = 1 + \alpha\cos(mx)$, the exact positive non-monotonic solution $u(x) = \rho(x)\theta(x)$ was obtained in [1] for some positive and strictly decreasing $\theta(x)$ such that $\theta(x) \to 0$ as $x \to \infty$. Furthermore, in [3] for more general class of inhomogeneous nonlinear Schrödinger equations:

$$u'' + (\mu - V(x))u = f(x, |u|^2)u, \qquad \text{(NLSE-general)}$$

some qualitative conditions on the functions $\mu - V(x)$ and $f(x, s)$ were given such that every positive solution $u(x)$ of (NLSE-general) has the strong non-monotonic behaviour, that is, $u'(x)$ changes sing infinitely many times as $x \to \infty$.

For the above exact solution of (NLSE-exact), where $u(x) - \theta(x) = \alpha\cos(mx)\theta(x)$, we especially conclude that $u(x) - \theta(x)$ changes sign infinitely many times as $x \to \infty$, which geometrically means that the graph of $u(x)$ crosses infinitely many times over the graph of $\theta(x)$. Therefore, one can say that (NLSE-exact) has a *positive relative oscillation* with respect to a given $\theta(x)$. Our aim is to establish the existence of a positive relative oscillation in (NLSE-general). As the first step in this direction, we propose an approach to this problem for the non-homogeneous second-order linear differential equation: $(p(x)u'(x))' + q(x)u(x) = e(x)$, motivated with some methods already published in [4].

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## Asymptotic analysis of the heat conduction problem in a dilated pipe


Marija Prša

University of Zagreb, Faculty of Graphic Arts

marija.prsa@grf.hr


We studied a heat conduction problem in a pipe filled with incompressible viscous fluid whose length varies depending on the fluid temperature. We assume that the longitudinal dilatation of the pipe is described by a linear heat expansion law, i.e. length of the pipe varies with the heat expansion coefficient.
In our previous work we have seen under which conditions there is a solution to the proposed problem and when is the solution unique.
In this work we constructed an approximation of the solution by means of an asymptotic expansion in powers of the abovementioned coefficient, followed by rigorous justification of the obtained model by proving the appropriate error estimate.
Finally, we studied a special case of a thin pipe, where we constructed the approximation of the solution by means of an asymptotic expansion in powers of the pipe radius. We achieved the solution approximation for the first two terms and subsequently justified the result by proving adequate error estimates.

This is a joint work with prof. Eduard Marušić-Paloka and prof. Igor Pažanin, Faculty of Science, University of Zagreb.

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## Convergence of a finite volume scheme for immiscible compressible two-phase flow in porous media

Mladen Jurak[1], Ivana Radišić[2]

Faculty of Science, University of Zagreb[1],
Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb[2]

jurak@math.hr[1], iradisic@fsb.hr[2]

A robust, industrial strenght simulator of compressible two-phase flow in porous media can be implemented by a finite volume method together with phase-by-phase upwind stabilisation. In this article we prove the convergence of such a scheme applied to a compressible two-phase flow with gravity and capillary terms. The proof relies on the concept of global pressure based on the total flux and on upwinding by the global pressure as a tool for deriving a discrete energy estimate, which has a central role in the convergence proof.

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## Regularity result for 3D incompressible fluid-rigid body interaction problem

Ana Radošević

Department of Mathematics, Faculty of Economics and Business, University of Zagreb

aradosevic@efzg.hr

We study a 3D fluid-rigid body interaction problem. The fluid flow is governed by 3D incompressible Navier-Stokes equations, while the motion of the rigid body is described by a system of ordinary differential equations called Euler equations for the rigid body. Our aim is to prove a generalization of the regularity result for weak solutions to the Navier-Stokes equations, which says that a weak solution that additionally satisfy Prodi-Serrin $L^r - L^s$ condition is smooth. This is a joint work with Boris Muha and Šarka Nečasová.

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## On the Darcy–Brinkman–Boussinesq flow through a thin channel with slightly perturbed boundary

Eduard Marušić–Paloka[1], Igor Pažanin[2], Marko Radulović[3]

Department of Mathematics, Faculty of Science, University of Zagreb[1,2,3]

emarusic@math.hr[1], pazanin@math.hr[2], mradul@math.hr[3]

This is a joint work with Prof. Eduard Marušić–Paloka and Prof. Igor Pažanin.

In this talk, we will investigate the effects of a small boundary perturbation on the non-isothermal fluid flow through a thin channel filled with porous medium. Starting from the Darcy–Brinkman–Boussinesq system of equations and employing methods of asymptotic analysis, we derive a higher-order effective model given by the explicit formulae. To observe the effects of the boundary irregularities, we numerically visualize the asymptotic approximation for the temperature, whereas the justification and the order of accuracy of the model is provided by the theoretical error analysis.

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## Spatiotemporal dynamics and reliable computations in recurrent spiking neural networks

Robert Rosenbaum

University of Notre Dame

Robert.Rosenbaum@nd.edu

Randomly connected networks of spiking neuron models provide a parsimonious model of neural variability, but are notoriously unreliable for performing computations. We show that this difficulty can be overcome by incorporating the well-documented dependence of connection probability on distance. Using a Fokker-Planck formalism, we show that spatially extended spiking networks exhibit symmetry-breaking, Turing-Hopf bifurcations to generate intricate spatiotemporal patterns. The resulting dynamics can be trained to perform dynamical computations using a reservoir computing approach.

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# Tensor-Krylov method for parametrized eigenvalue problems

Koen Ruymbeek[1], Karl Meerbergen[2], Wim Michiels[3]
KU Leuven
Koen.Ruymbeek@kuleuven.be[1], Karl.Meerbergen@kuleuven.be[2], Wim.Michiels@kuleuven.be[3]

In this talk, we consider the parameter-dependent eigenvalue problem

$$\mathbf{A}(\omega)\mathbf{x}(\omega) = \lambda(\omega)\mathbf{x}(\omega), \quad \omega \in \Omega, \quad \mathbf{A} : \Omega \to \mathbb{R}^{n\times n} : \omega \mapsto \sum_{i=1}^{m} f_i(\omega)\mathbf{A}_i \tag{2}$$

on a compact parameter set $\Omega \subset \mathbb{R}^d$ with separable functions $f_i, i = 1, 2, \ldots, m$ where $m$ is small relative to the dimension. We look for one particular eigenvalue $\lambda_1$ over the parameter space, e.g. $\lambda_1$ is the dominant eigenvalue. We assume that the wanted eigenvalue converges for any $\omega$ in a few iterations of Arnoldi. In practice this means that $\lambda_1$ should be a well separated extreme eigenvalue of $\mathbf{A}(\omega)$. We assume that calculating $\lambda_1$ for many values of $\omega$ is infeasible. A requirement for our method is that the desired eigenvectors do not significantly change with $\omega$, i.e. they span a space which can well be approximated by a low-dimensional space.

In our algorithm we apply the Residual Arnoldi (RA) algorithm simultaneously on a set of parameter values obtained by discretising $\Omega$. The RA algorithm is an adaptation of the classical Arnoldi algorithm for calculating eigenvalues where in each iteration the residual of the Ritz approximation of the desired eigenpair is added to the subspace. We then calculate an estimation of the desired eigenvector for each parameter values within this subspace. It is proven that this is far more robust against introduced error than the classical Arnoldi algorithm.

Instead of adding the residuals for all parameter values to the subspace, we make a low rank approximation of the space of residuals, and we only use the corresponding basis vectors to extend the subspace. Therefore our method is an example of a Tensor-Krylov method.

A practical issue is the growth of the dimension of this subspace as we add multiple vectors in each iteration. As the dimension of this subspace is limited for efficiency reasons, we need to restart the subspace often which inevitably causes loss in accuracy. The novelty of this research lies in the optimal use of this subspace and is twofold. Firstly, we observed that a large error in the low-rank approximations is allowed without harming the convergence, meaning we can do more iterations without restarting. In the talk we explain this observation with theory as well as with practical examples. Secondly, we look at how the subspace is restarted. On the one hand we need to limit the dimension of the restarted subspace but on the other hand we do not want to lose much accuracy. In Arnoldi-like algorithms, the idea in the restarting method is to delete the part of the subspace which is associated with eigenvalues far away from $\lambda_1$. As we have one subspace for all parameter values, this subspace would still be too large. We choose only to keep the subspace which is associated with our current Ritz vector for all parameter values. We finish the talk with some words why we choose Tensor-Trains as tensor format when having more than one parameter and show some numerical experiments.

## Weak-strong uniqueness for fluid-structure interactions

S. Schwarzacher, M. Sroczinski
Charles University
`schwarz@karlin.mff.cuni.cz`

We discuss weak-strong uniqueness and stability results for the motion of a two or three dimensional fluid governed by the Navier-Stokes equation interacting with a flexible, elastic plate of Koiter type. The uniqueness result is a consequence of a stability estimate where the difference of two solutions is estimated by the distance of the initial values and outer forces. For that we introduce a methodology that overcomes the problem that the two (variable in time) domains of the fluid velocities and pressures are not the same. The estimate holds under the assumption that one of the two weak solutions possesses some additional higher regularity. The additional regularity is exclusively requested for the velocity of one of the solutions resembling the celebrated Ladyzhenskaya-Prodi-Serrin conditions in the framework of variable domains.

## On the stability of a mixed finite volume – finite element method for compressible fluid-structure interaction

Bangwei She
Charles University
Institute of Mathematics of the Czech Academy of Sciences
`she@math.cas.cz`

We propose a mixed finite volume – finite element method for the approximation of compressible fluids interacting with an elastic structure. We show positivity of the discrete density, existence of a numerical solution, and the energy stability of the numerical solution. The results are based on the joint works with Boris Muha (Uni-Zagreb), Sebastian Schwarzacher (Charles-Uni), and Srdjan Trifunovic (Shanghai Jiaotong Uni).

# Application of the domain decomposition method in the variational data assimilation problem: description of two approaches

Natalia Lezina[1], Tatiana Sheloput[2]

Marchuk Institute of Numerical Mathematics of the Russian Academy of Sciences[1,2]

lezina@phystech.edu[1], sheloput@phystech.edu[2]

The use of domain decomposition methods in variational data assimilation is a relatively new and uninvestigated area, but the development of this topic may be promising for some scientific problems. The use of domain decomposition method (DDM) allows to reduce the solution process in the original domain to alternate solving the problem in subdomains, possibly having a simpler form, or apply different models in them. Particularly, it is possible to use DDM to combine models developed for individual phenomena at specific scales. Application of DDM in variational data assimilation provides an opportunity for creating parallel algorithms. Moreover, observational data could be available not in the whole modelling area but only in some subdomain in which variational data assimilation procedure may be considered. In this case the application of domain decomposition methods seems to be effective.

In the present study the DDM based on the theory of adjoint equations and optimal control is applied to the problems of variational assimilation of the sea level and barotropic velocity. The simple open water area model based on the linearised shallow water equations is considered. The original domain is divided into two subdomains by introducing inner liquid boundary. The boundary conditions on the open boundary and on the inner liquid boundary are formulated by introducing additional unknown functions. The following inverse problem is formulated: find the functions of solutions in the subdomains and unknown boundary functions, satisfying the systems of equations in subdomains and additional “closure conditions”. The inverse problem is considered in a weak form. To investigate the problem it is rewritten in the operator form. The Tikhonov regularization method is applied to solve the ill-posed inverse problem. The two approaches to problem solution are described, iterative algorithms are proposed, unique and dense solvability of the problem is proved, and convergence of the iterative algorithms is studied. The results of the numerical experiments are presented and discussed.

The work was supported by the Russian Science Foundation (project 19-71-20035, the numerical experiments) and by the Russian Foundation for Basic Research (project 19-01-00595, the study of the problem and formulation of the iterative algorithms).

## Solenoidal extensions in domains with obstacles: explicit bounds and applications to Navier-Stokes equations

Gianmarco Sperone
Charles University in Prague
sperone@karlin.mff.cuni.cz

We introduce a new method for constructing solenoidal extensions of fairly general boundary data in (2d or 3d) cubes that contain an obstacle. This method allows us to provide *explicit* bounds for the Dirichlet norm of the extensions. It runs as follows: by inverting the trace operator, we first determine suitable extensions, not necessarily solenoidal, of the data; then we analyze the Bogovskii problem with the resulting divergence to obtain a solenoidal extension; finally, by solving a variational problem involving the infinity-Laplacian and using ad hoc cutoff functions, we find explicit bounds in terms of the geometric parameters of the obstacle. The natural applications of our results lie in the analysis of inflow-outflow problems, in which an explicit bound on the inflow velocity is needed to estimate the threshold for uniqueness in the stationary Navier-Stokes equations and, in case of symmetry, the stability of the obstacle immersed in the fluid flow. This is a joint work with Ilaria Fragalà and Filippo Gazzola (Politecnico di Milano).

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## New numerical algorithm for deflation of infinite and zero eigenvalues and full solution of quadratic eigenvalue problems

Zlatko Drmač[1], Ivana Šain Glibić[2]
University of Zagreb, Faculty of Science, Department of Mathematics[1,2]
drmac@math.hr[1], ivanasai@math.hr[2]

In this talk we will present a new method for computing all eigenvalues and eigenvectors of the quadratic eigenvalue problem $(\lambda^2 M + \lambda C + K)x = 0$. It is an upgrade of the `quadeig` algorithm by Hammarling, Munro and Tisseur, which attempts to reveal and remove by deflation certain number of zero and infinite eigenvalues before QZ iterations.

Proposed modifications of the `quadeig` framework are designed to enhance backward stability and to make the process of deflating infinite and zero eigenvalues more numerically robust. Using an upper triangular version of the Kronecker canonical form proposed algorithm deflates additional infinite and zero eigenvalues, in addition to those conducted from the rank of the corresponding coefficient matrices $M$ and $K$.

Finally, we present examples which confirms superior numerical performances of the proposed method.

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## Optimal design of stents

Josip Tambača
University of Zagreb
tambaca@math.hr

Stents are elastic meshes that are implanted into blood vessels to keep them open. Their mechanical properties are very dependent on very complex mesh structure. Thus on the market there are several different producers with different designs of stents. In this talk a possible attempt in order to obtain optimal design of stents using one dimensional curved rod model for stents will be presented. The focus will be on two optimization problems, the first with respect to the thickness of stent's struts (the edges in the mesh) and the second with respect to certain changes of geometry, but keeping the topology of the mesh fixed.

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## Efficient approximation of novel residual bounds for parameter dependent quadratic eigenvalue problem

Matea Ugrica, Zoran Tomljanović, Ninoslav Truhar
Department of Mathematics, Josip Juraj Strossmayer University of Osijek, Croatia
{mpuvaca,ztomljan,ntruhar}@mathos.hr

We consider the perturbation theory for parameter dependent quadratic eigenvalue problems (PQEP)

$$(\lambda(v)^2 M(v) + \lambda(v) D(v) + K(v))x(v) = 0, \quad x(v) \neq 0,$$

where $M(v)$, $D(v)$, $K(v)$ are Hermitian matrices of order $n$ and $M(v)$, $K(v)$ are positive definite for all parameters $v$ encoded in vector $v$.

We present new perturbation bounds between individual unperturbed and perturbed eigenvectors for PQEP. Since the new bound, but also many other relevant bounds for eigenvectors, contain gap functions, which depend on unperturbed and perturbed eigenvalues, in the second part we present upper bound for a different gap functions. These upper bounds are based on the first order approximation of eigenvalues involved in gap functions. The quality of the obtained bounds have been illustrated in numerical experiments.

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## Perturbations of invariant pairs of polynomial eigenvalue problems

Ninoslav Truhar
Department of Mathematics, University of Osijek, Croatia
ntruhar@mathos.hr

Let

$$P_n(\lambda)x \doteq \lambda^n A_n + \lambda^{n-1} A_{n-1} + \ldots + \lambda A_1 + A_0\,,$$

be given matrix polynomial, where $A_i \in \mathbb{C}^{n\times n}$ are Hermitian $A_i = A_i^*$, $i = 1, \ldots n$.

This talk is concerned with the problem of perturbations of the invariant pair $(X, S) \in \mathbb{C}^{n\times k} \times \mathbb{C}^{k\times k}$ of the considered polynomial eigenvalue problem

$$\mathbf{P}(X, S) \doteq A_n X S^n + A_{n-1} X S^{n-1} + \ldots + A_1 X S + A_0 X = 0$$

where $A_i \in \mathbb{C}^{n\times n}$ are Hermitian $A_i = A_i^*$, $i = 1, \ldots n$. The corresponding perturbed problem is

$$\widetilde{\mathbf{P}}(\widetilde{X}, \widetilde{S}) \doteq \widetilde{A}_n \widetilde{X} \widetilde{S}^n + \widetilde{A}_{n-1} \widetilde{X} \widetilde{S}^{n-1} + \ldots + \widetilde{A}_1 \widetilde{X} \widetilde{S} + \widetilde{A}_0 \widetilde{X} = 0$$

where $\widetilde{A}_i \in \mathbb{C}^{n\times n}$, are Hermitian $\widetilde{A}_i = A_i^*$, for $i = 1, \ldots n$.

We present a novel upper and lower bound for the $\|\sin\Theta(X, \widetilde{X})\|_F$ and $\|\cos\Theta(X, \widetilde{X})\|_F$, where $\Theta(X, \widetilde{X})$ is a matrix of canonical angles between column subspaces spanned with $X$ and $\widetilde{X}$, respectively.

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## Frequency-weighted damping via nonsmooth optimization and fast computation of QEPs with low-rank updates

Matea Ugrica[1], Nevena Jakovčević Stor[2], Tim Mitchell[3], Zoran Tomljanović[4]
Department of Mathematics, University J.J. Strossmayer[1], Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, University of Split[2], Max Planck Institute for Dynamics of Complex Technical Systems[3], Department of Mathematics, University J.J. Strossmayer[4]
mugrica@mathos.hr[1], nevena@fesb.hr[2], mitchell@mpi-magdeburg.mpg.de[3], ztomljan@mathos.hr[4]

We consider a frequency-weighted damping optimization in vibrating systems described by second-order differential equation. The main problem is to determine the best damping positions and viscosities such that eigenvalues of damped system are not close to undesirable areas. We achieve this by first formulating two novel nonsmooth constrained optimization problems which, when solved, both damp undesirable frequency bands and maintain stability of the system. These approaches also allows us to weight which frequency bands are most critical to damp. Since damping optimization process requires solutions of a sequence of related QEPs (with different damping parameters), in this work we provide a fast solution of the QEP based on the method for eigendecomposition of complex symmetric diagonal-plus-rank-one matrix. The methodology proposed here provides a significant acceleration of the damping optimization process and the numerical experiments confirm the efficiency of our new approach.

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## Local stability of optimal designs in transmition problems on a ball

Marko Vrdoljak

Department of Mathematics, Faculty of Science, University of Zagreb

marko@math.hr

We study energy functional of the transmition problem on a thermal conductive ball with two isotropic phases. The aim is to find a distribution of materials which maximizes the functional. By using the first and the second order shape derivatives we study the question whether critical stable shapes are local maxima for smooth perturbations. The main problem is to concord different norms: the one in coercivity condition for a critical shape and the other describing its small neighborhoods.

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