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Contents

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80. Michal Žeravý, *Slovak University of Technology in Bratislava*

Programme

Opening and closing, as well as all morning and plenary lectures take place in the conference room **Kastrum.** Afternoon lectures take place in conference rooms **Verige** and **Saline.** Poster sessions take place in the **lobby** in front of the conference room Kastrum.

Monday, September 5th

08:50 - 09:00	Opening	
09:00 - 09:45	**Plenary: Zunino**	
09:45 - 10:35	Poster blitz #1	
10:35 - 11:05	Coffee break	
11:05 - 11:50	**Plenary: Kukavica**	
11:50 - 12:15	Tuffaha	
12:15 - 13:45	Lunch	
13:45 - 14:10	Tambača	Mácha
14:10 - 14:35	Bujanović	Bosner T.
14:35 - 15:35	Coffee break + Poster session #1	
15:35 - 16:00	Bosner N.	Vrdoljak
16:00 - 16:25	Ljulj	Mlinarić

Poster session #1 : Alnafisah, Češík, Ćurković, Gosea, He, Kalousek, Kunštek, Lacmanović, Nepal, Perković, Persson, Pilj Vidaković, Prša, Radošević, Séguin, Strössner, Zavalani.

Tuesday, September 6th

09:00 - 09:45	**Plenary: Simoncini**	
09:45 - 10:35	Poster blitz #2	
10:35 - 11:05	Coffee break	
11:05 - 11:30	Schwarzacher	
11:30 - 11:55	Marušić-Paloka	
11:55 - 12:20	Nečasová	
12:20 - 13:45	Lunch	
13:45 - 14:10	She	Slapničar
14:10 - 14:35	Ceruti	Kollár
14:35 - 15:00	Chemetov	Vrbaški
15:00 - 16:00	Coffee break + Poster session #2	
16:00 - 16:45	**Plenary: Lasiecka**	

Poster session #2 : Carević, Das, Galić, Grbac, Guido, Jankov, Lam, Ma, Ožvat, Przybilla, Raveendran, Rukavina, Skříšovský, Vojnović, Žáková, Žeravý.

Wednesday, September 7th

09:00 - 09:45	**Plenary: Heltai**
09:45 - 10:10	Richter
10:10 - 10:35	Neuss-Radu
10:35 - 11:05	Coffee break
11:05 - 11:50	**Plenary: Horváth**
11:50 - 12:15	Bukač
12:15 - 12:40	Grubišić
12:40 - 14:00	Lunch
14:00 - 15:30	EU-MATHS-IN Round Table*
15:30 - 19:00	Free time
19:30 - 00:00	Conference dinner

*Moderator: Z. Horváth (room Saline)

Thursday, September 8th

09:00 - 09:45	**Plenary: Benner**	
09:45 - 10:10	Kressner	
10:10 - 10:35	Drmač	
10:35 - 11:05	Coffee break	
11:05 - 11:50	**Plenary: Gugercin**	
11:50 - 12:15	Tomljanović	
12:15 - 13:45	Lunch	
13:45 - 14:10	Webster	Burazin
14:10 - 14:35	Rosenbaum	Bukal
14:35 - 15:00	Vlah	Erceg
15:00 - 15:30	Coffee break	
15:30 - 15:55	Kuan	Kučera
15:55 - 16:20	Valášek	Mehlmann

Friday, September 9th

09:00 - 09:45	**Plenary: Pileckas**
09:45 - 10:10	Lazar
10:10 - 10:35	Veselić
10:35 - 11:05	Coffee break
11:05 - 11:30	Trifunović
11:30 - 11:55	Antonić
11:55 - 12:20	Pokorný
12:20 - 12:30	Closing
12:30 - 14:00	Lunch

Invited Talks

Data-enhanced Reduced-order Modeling for Dynamical Systems

Peter Benner[1], Pawan Goyal[2], Igor Pontes Duff[3]
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The construction of surrogate models for faster simulation, optimization and control of dynamical systems is an active research area in many areas of the computational sciences and engineering. Model order reduction (MOR) is the prevailing technique to compute a reduced-order system as a surrogate for a high-dimensional (full-order) system, using often some form of (Petrov-)Galerkin projection. In this talk, we will discuss a technique for computing dominant subspaces related to structured parametric systems. Here, dominance relates to the "most reachable and observable" states of the full-order system. We show that these subspaces can be obtained using interpolation techniques and oversampling. Moreover, available data from measurements in frequency domain can be incorporated into the resulting algorithm. Extensions to nonlinear systems are also discussed. We illustrate the performance of the derived model reduction framework using several numerical benchmark examples.

Model reduction and data-driven modeling for power networks

Serkan Gugercin
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Power networks are complex and large-scale systems in operation. Simulation of the corresponding models are computationally expensive and demand unmanageable levels of storage in dynamic simulation or trajectory sensitivity analysis. Therefore, reducing the order of power network dynamics is of great importance especially for real time applications.

Dynamical systems representing power networks have inherent structure reflecting underlying physics. When constructing reduced-order models for these systems it is vital to retain this structure so that the reduced models can be interpreted

as physically meaningful surrogates. In this talk, we first develop a projection-based structure-preserving system-theoretic model reduction framework for nonlinear power grid networks. Via a lifting transformation, we convert the original nonlinear system to an equivalent quadratic nonlinear model. This equivalent representation, in turn, allows us to employ a wide range of input-independent model reduction techniques that also retain critical nonlinear structural features of the original system.

Next we consider complementary data-driven approaches to power network modeling where access to internal dynamics are not available. As in the projection-based framework, when modeling a dynamical system from data, it is important to retain the underlying physical structures. This is the framework we consider for modeling power networks from data where the structure to preserve naturally results from nonlinear swing equations. Using the time-domain state-snapshot data, we develop approaches to constructing structured surrogates for nonlinear swing equations. Moreover, for the real-time power system event analysis, we introduce a selection procedure for locating power system events from real-time data based on the Discrete Empirical Interpolation Method.

A Reduced Lagrange Multiplier framework for the simulation of multi-dimensional and multi-physics coupled problems

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Real life multi-phase and multi-physics problems coupled across different scales present outstanding challenges, whose practical resolution often require unconventional numerical methods. When the intrinsic dimensions of the immersed phase are orders of magnitude smaller than the system scale, the numerical simulation of a fully resolved geometrical model becomes unfeasible. Yet, complex phenomena that happen at the smaller scale have an effect on the global behavior of the material, and one is often interested in the coupling between the two phases without homogenising away the immersed phase. Non-matching and immersed methods can be used to enable the mesoscale resolution of problems with embedded structures, and they provide a natural way to perform geometric dimensionality reduction. In this talk I will discuss how to leverage immersed finite element methods and (distributed) Lagrange multipliers to formulate coupled multi-physics and multi-dimensional problems in a mathematically consistent way, analyse the model error deriving from the dimensionality reduction, and provide some relevant numerical examples.

Overview and invitation to EU-MATHS-IN ongoing actions: Open Desk, short and long-term projects, collaboration with technology platforms, and a digital twin use case analyses

Zoltán Horváth
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EU-MATHS-IN is the network of 20 European National Networks of industrial mathematics to create and operate an industrial mathematics ecosystem for services and innovation actions for all industries, at the European level. The activities in this ecosystem include strategic collaboration with the European decision makers responsible for research and innovation work programmes, collaboration with the existing technology platforms of industries, and the operation of a service platform. In the talk, we provide details on the results that EU-MATHS-IN has achieved. In addition, we introduce the concrete ongoing actions of EU-MATHS-IN, which are open to each member. Thus, we shall provide explanation and connection options for the actions as follows:

- Open Desk service platform, which attracts problems of EU industries and matches the interested research group in order to create a project for solution,
- building project consortia for an open EU research and innovation calls in the topics of creating new algorithms for HPC,
- building project consortia for an open EU call to create a long-standing infrastructure for the industrial mathematics ecosystem,
- collaboration opportunities with the digital platforms of the European innovation ecosystem within the TransContinuum initiative, in particular
- building a team that drafts strategic papers for the Destination Earth programme,
- introduce a digital twin use case that is analyzed in a strategic collaboration with other platforms; here we shall provide details of the software for model order reduction of compressible fluids and some arisen mathematical challenges,
- drafting the Strategic Research Agenda for the industrial mathematics ecosystem, which is EU-MATHS-IN's "business card" for collaborations.

A free boundary inviscid model of flow-structure interaction

Igor Kukavica[1], Amjad Tuffaha[2]
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We address a system describing interaction of an incompressible inviscid fluid, modeled by the Euler equations, and an elastic plate, represented by a fourth-order hyperbolic PDE. We provide a priori estimates for the existence of solutions with a sharp regularity for the Euler initial data and construct solutions with the regularity construct solutions with the fluid initial data in H^r, where $r \geq 3$.

On weak/strong attractor for a 3-D Structural Acoustic Interaction with Kirchhoff-Boussinesq Interface

Irena Lasiecka
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3-D structural acoustic interaction with an elastic wall modeled by a nonlinear Kirchhoff-Boussinesq dynamic plate equation is considered. The model arises within a context of noise suppression/control in an acoustic cham- ber surrounded by rigid and elastic walls. The attenuation/control of the noise takes place on an elastic wall which by itself is modeled by plate equation with supercritical restoring forces and Boussinesq nonlinear term leading o a potential finite time blow up of solutions. In addition to Boussinesq source [resulting from a strong nonlinear coupling between vertical and transverse shear effects in the Midlin Timoshenko system], plate dynamics is ignited by an acoustic pressure resulting from wave propagation in an acoustic chamber. The key to the result is a boundary feedback control applied on the elastic wall. Hadamard wellposedness of weak and strong solutions will be discussed along with the long time behavior of the corresponding solutions. From mathematical point of view, the interesting features are the lack of dissipativity and the lack of compactness resulting from nonlinear plate effects and lack of time reversibility resulting from boundary control of acoustic waves. As it is known these features are fundamental in the theory of dynamical systems arising in hyperbolic like dynamics. To contend with the difficulty, a new "hybrid" method suitable for handling long time behavior of coupled dynamics with "poor communication" will be presented.

Existence of non-stationary Poiseuille type solutions under minimal regularity assumptions

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Existence and uniqueness of a solution to the non-stationary Navier-Stokes equations having a prescribed flow rate (flux) in the infinite cylinder $\Pi = \{x : (x_1, x_2) \in \sigma, x_3 \in (-\infty, +\infty)\}$, where σ is bounded domain in R^2, are proved. It is assumed that the flow rate $F(t)$ is an element of $L^2(0,T)$ and the initial data $u_0 = (0, 0, u_{0n})$ is an element of $L^2(sigma)$. The non-stationary Poiseuille solution has the form $u(x,t) = (0, 0, U(x',t))$, $p(x,t) = -q(t)x(n) + p_0(t)$, where $(U(x',t), q(t))$ is a solution of an inverse problem for the heat equation with a specific over-determination condition. Under the above regularity assumptions the solution of the problem does not have the usual for parabolic problems regularity: it is much weaker.

On time-dependent matrix-oriented differential problems

Valeria Simoncini

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Nonlinear time-dependent differential equations can be treated following two quite different paradigms for time discretization. The classical way consists of using a time marching iteration that appropriately deals with the stiff and nonstiff portions of the nonlinear terms stemming from the space discretization. An alternative way is to treat time as if it were an additional space variable: all-at-once or parallel-in-time approaches, and space-time variational formulations belong to this family of discretizations. In this talk we survey a few developed techniques from both paradigms, that also exploit recent matrix-oriented approaches for the associated linear algebra phase.

A deep learning approach to reduced order modeling of parameter dependent partial differential equations

Paolo Zunino[1], Nicola-Rares Franco, Andrea Manzoni, Stefania Fresca

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In the context of parametrized PDEs, Reduced Order Models (ROMs) allow for an efficient approximation of the parameter-to-solution map, which is extremely useful whenever dealing with expensive many-query routines such as constrained optimization, sensitivity analysis and uncertainty quantification. Recently, motivated by the limitations of classical approaches such as the Reduced Basis method, many authors have been considering the use of Neural Networks (NNs) and Deep Learning techniques for building non- intrusive ROMs, see for example [1,2,3,4,5]. We tackle the problem of developing a constructive approach based on Deep Neural Networks for the efficient approximation of the parameter-to-solution map of PDEs [1]. Our work is based on the use of deep autoencoders, which we employ for encoding and decoding a high-fidelity approximation of the solution manifold. To provide guidelines for the design of deep autoencoders, we consider a nonlinear version of the Kolmogorov n-width over which we base the concept of a minimal latent dimension. We show that the latter is intimately related to the topological properties of the solution manifold, and we provide theoretical results with particular emphasis on second order elliptic PDEs, characterizing the minimal dimension and the approximation errors of the proposed approach. We also study the expressivity of the deep neural networks with respect to the solution manifold and we present some results that illustrate the connection between the expressivity and the complexity of the networks. Furthermore, we discuss its generalization to PDEs involving complex domains, leveraging on the idea of sparsifying the network architecture by means of information about the topology of the computational mesh used for the full order model. We call this approach the Mesh Informed Neural Networks [5]. The extension to time dependent parametrized PDEs is also briefly discussed [4]. Finally, the theory presented is supported by numerical experiments, where we compare the proposed approach with classical POD-Galerkin ROMs.

References:

[1] N. R. Franco, A. Manzoni, and P. Zunino. *A deep learning approach to reduced order modelling of parameter dependent partial differential equations*. To appear on Mathematics of Computation (2021) arXiv:2103.06183

[2] J.S. Hesthaven, S. Ubbiali, *Non-intrusive reduced order modeling of nonlinear problems using neural networks*, Journal of Computational Physics,V olume 363, 2018,

[3] Kutyniok, G., Petersen, P., Raslan, M. and Schneider, R., *A Theoretical Analysis of Deep Neural Networks and Parametric PDEs*, Constructive Approximation, (2021) 1-53

[4] S. Fresca and A. Manzoni. *POD-DL-ROM: enhancing deep learning-based reduced order models for nonlinear parametrized PDEs by proper orthogonal decomposition.* Computer Methods in Applied Mechanics and Engineering, (2022) 388:114181

[5] Franco, N.R., Manzoni, A., Zunino, P. *Learning Operators with Mesh-Informed Neural Networks* (2022) arXiv:2203.11648

Contributed Talks

Comparison between Milstein and exact coupling methods using MATLAB for a particular two-dimensional stochastic differential equation

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We compare Milstein and exact coupling methods for the strong approximation of solutions to stochastic differential equations (SDE), which are driven by Brownian motion. Both of these methods attain an order one convergence under the non-degeneracy assumption of the diffusion term for the exact coupling method. We also compare their implementation using MATLAB. A particular two-dimensional SDE is used in the implementation for comparing their results. Moreover, the performance of both methods and the amount of time required to obtain the result are also analysed. It is interesting to mention that this comparison is very important in several areas, such as stochastic analysis, financial mathematics and some physical applications.

Localisation principle for microlocal defect functionals and small-amplitude homogenisation

Nenad Antonić

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Microlocal defect functionals encompass various objects, like H-distributions, one-scale H-measures etc., which extend the H-measures (also called microlocal defect measures) and semiclassical measures (or Wigner measures). The localisation principle (extending the compactnes by compensation theory) for such objects will be discussed, as well as its applications to small-amplitude homogenisation.

Parallel Implementations of Joint Approximate Diagonalization

Nela Bosner

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Our goal is to solve efficiently the following problem: given a set of symmetric matrices $A^{(1)}$, $A^{(2)}$, ...,$A^{(m)} \in \mathbb{R}^{n\times n}$, find orthogonal or nonsingular $X \in \mathbb{R}^{n\times n}$ such that

$$A^{(p)} = X \cdot D^{(p)} \cdot X^T, \quad \text{for all } p = 1, \ldots, m,$$

where $D^{(p)}$ are either diagonal (exact diagonalization), or as diagonal as possible according to some criterion (approximate diagonalization) in case of noisy input data. We were focused on approach which offers opportunity for efficient parallel implementation, especially for larger dimension n and large number of matrices m. This approach is based on Riemannian optimization by conjugate gradient method on two matrix manifolds: the Stiefel manifold (orthogonal group), when diagonalizing matrix X needs to be orthogonal, and the oblique manifold (matrices with unit norm columns), otherwise. The conjugate gradient optimization method requires solution of 1-dimensional optimization problem in each iteration, has superlinear convergence, and is very simple for implementation and suitable for parallelization. The objective function on both manifolds is

$$F(X) = \frac{1}{2}\sum_{p=1}^{m} \|\mathrm{Off}(X^T A^{(p)} X)\|_F^2,$$

where for $B = [b_{ij}]$, $\mathrm{Off}(B) = B - \mathrm{Diag}(B)$, and $\mathrm{Diag}(B) = \mathrm{diag}(b_{11}, \ldots, b_{mm})$. Diagonalization or approximate diagonalization is represented by minimization problem

$$\min_{X\in\mathcal{M}} F(X), \qquad \text{where } \mathcal{M} \text{ is appropriate manifold.}$$

In our implementation of the method, all basic steps, such as: evaluation of $F(X)$, $\mathrm{grad}F(X)$, and $\mathrm{Hess}F(X)$, computing geodesic and solving 1-dimensional optimization problem, is explicitly modified and parallelized in order to decrease operation count. Numerical experiments confirmed that our modified implementations of the conjugate gradient method on two matrix manifolds are more efficient than the nonparallel versions, in particular for large n and m.

Generalizations of CCC–Schoenberg operators of higher order

Tina Bosner
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We know that Schoenberg operators associated with Canonical Complete Chebyshev splines have quadratic convergence. The question that arises is: is it possible to construct a generalization of the Schoenberg operators, that would approximate some function with higher order. We propose two kinds of such operators. The first one is based on dual functionals, and the second one is inspired by the operator used by de Boor for polynomial splines, which uses only the values of a given function at certain points. For both of them we show their convergence rate, and illustrate them with few examples.

Iterative refinement of Schur decompositions

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The Schur decomposition of a square matrix A is an important intermediate step of state-of-the-art numerical algorithms for addressing eigenvalue problems, matrix functions, and matrix equations. This talk is concerned with the following task: Compute a (more) *accurate* Schur decomposition of A from a given *approximate* Schur decomposition. This task arises, for example, in the context of parameter-dependent eigenvalue problems and mixed precision computations. We have developed a Newton-like algorithm that requires the solution of a triangular matrix equation and an approximate orthogonalization step in every iteration. We prove local quadratic convergence for matrices with mutually distinct eigenvalues and observe fast convergence in practice. In a mixed low-high precision environment, our algorithm essentially reduces to only four high-precision matrix-matrix multiplications per iteration. When refining double to quadruple precision, it often needs only 3–4 iterations, which reduces the time of computing a quadruple precision Schur decomposition by up to a factor of 10–20.

Adaptive time-stepping methods for fluid-structure interaction problems

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In realistic flow problems described by partial differential equations (PDEs), where the dynamics are not known, or in which the variables are changing rapidly, the robust, adaptive time-stepping is central to accurately and efficiently predict the long-term behavior of the solution. This is especially important in the coupled flow problems, such as the fluid-structure interaction (FSI), which often exhibit complex dynamic behavior. While the adaptive spatial mesh refinement techniques are well established and widely used, less attention has been given to the adaptive time-stepping methods for PDEs. We will discuss a novel, adaptive, partitioned numerical method for FSI problems. The time integration in the proposed methods is based on the refactorized Cauchy's one-legged 'theta-like' method, which consists of a backward Euler method, where the fluid and structure sub-problems are sub-iterated until convergence, followed by a forward Euler method. The bulk of the computation is done by the backward Euler method, as the forward Euler step is equivalent to (and implemented as) a linear extrapolation. We will present the numerical analysis of the proposed method showing linear convergence of the sub-iterative process and unconditional stability. The time adaptation strategies will be discussed. The properties of the method, as well as the selection of the parameters used in the adaptive process, will be explored in numerical examples.

Long time behavior of weak solutions to sixth-order thin-film equations

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We prove convergence of nonnegative weak solutions of sixth-order thin-film equations to the constant steady state in the energy norm at an algebraic rate. The result is based on a new functional inequality which acts as an energy – energy dissipation inequality for sixth-order thin-film equations. This is a joint work with S. Ulusoy (AURAK).

Numerical method for compliance minimization in 3D linearized elasticity

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In optimal design problems the goal is to find the arrangement of given materials which minimizes (or maximizes) a particular (integral) functional, under constraints on the amount of materials and PDE constraints that underlay involved physics. As such problems usually do not have a solution, a typical way to deal with them is by performing a relaxation. We are interested in the framework of the linearized elasticity, and the relaxation is performed by the homogenization method, in which one extends the set of classical materials by allowing that original materials are mixed on a very small scale, and thus obtaining so called composite materials. Mathematically, composite materials are homogenized limits of a sequences of classical materials.

One of the most prominent problems in structural mechanics is to describe all possible composites (known as the G-closure set) that can be achieved by mixing two materials (with or without prescribed ratio). Unfortunately, this "G-closure problem" is still an open problem, even for elastic composites made by mixing two isotropic (linearly) elastic phases, which makes corresponding optimal design problem highly nontrivial, as the set over which the optimization is performed remains unknown.

Luckily, for the compliance minimization problem, the relaxation can be performed over sequential laminates instead of the set of all composites, and the necessary conditions of optimality in this case can easily be derived. This enables a development of an optimality criteria method for finding an approximate solution. However, in order to implement this method one needs a precise knowledge of Hashin-Shtrikman bounds, since they naturally arise in the optimality conditions in the sense that the optimal design locally (in a given point) saturates the appropriate energy bound. Hashin-Shtrinkman energy bounds are probably the most prominent representative among various bounds on the set of all composite materials, which have imposed themselves as the benchmark against which most experimental results are compared. While explicit calculation of bounds in two space dimensions has been done three decades ago, the procedure in three space dimensions remained an open problem.

We shall briefly present an explicit calculation of Hashin-Shtrikman energy bounds in three space dimensions and use this new results for implementation of the optimality criteria method in three-dimensional compliance minimization problem. We shall finish with numerical examples and discussion on further perspectives yielded by these result, such as application in the modelling of the damage accumulation, as well as the coherent phase transitions.

This is joint work with Ivana Crnjac and Marko Vrdoljak.

Analyzing different versions of randomized (G)SVD for regularization of large scale discrete inverse problems

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Singular value decomposition (SVD) and generalized singular value decomposition (GSVD) can have an important role when regularizing an ill-posed discrete inverse problem. Since the calculation of (G)SVD for large-scale problems is time consuming, methods that can be used to accelerate it are of particular interest. Recently, research using randomized algorithms to speed up the computation of (G)SVD have been widely reported. Accordingly, existing regularization methods have been modified to use randomized (G)SVD. However, their efficiency may be questionable when regularizing an inverse problem in which the singular values of the operator matrix decay gradually without a significant gap. This is very common in application, usually when the problem arise from discretization of a Fredholm integral equation. In this talk we analyze the application of different versions of randomized (G)SVD to inverse problem in which the singular values decay gradually without a significant gap and offer some improvements for their efficiency.

Rank-adaptive time-integrator for dynamical low-rank approximation

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In the present contribution, a rank-adaptive integrator for the dynamical low-rank approximation of matrix and tensor differential equations is presented. The fixed-rank integrator recently proposed by two of the authors is extended to allow for an adaptive choice of the rank, using subspaces that are generated by the integrator itself. The integrator first updates the evolving bases and then does a Galerkin step in the subspace generated by both the new and old bases, which is followed by rank truncation to a given tolerance. It is shown that the adaptive low-rank integrator retains the exactness, robustness and symmetry-preserving properties of the previously proposed fixed-rank integrator. Beyond that, up to the truncation tolerance, the rank-adaptive integrator preserves the norm when the differential equation does, it preserves the energy for Schrodinger equations and Hamiltonian systems, and it preserves the monotonic decrease of the functional in gradient flows.

Injection-suction control for Navier-Stokes equations with slippage

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We consider a velocity tracking problem for the Navier-Stokes equations in a 2D-bounded domain. The control acts on the boundary through a injection-suction device and the flow is allowed to slip against the surface wall. We show the existence of an optimal solution for the problem. Also we discuss the uniqueness result. Joint work with F. Cipriano (Universidade Nova de Lisboa, Portugal).

References:

[1] Chemetov N.V., Cipriano F., *Injection-Suction Control for Two-Dimensional Navier–Stokes Equations with Slippage*, SIAM Journal on Control and Optimization, **56** 2, 1253-1281 (2018).

[2] Chemetov N.V., Cipriano F., *Optimal control for two-dimensional stochastic second grade fluids.* Stochastic Processes and their Applications, **128** 8, 2710-2749 (2018).

[3] Chemetov N.V., Cipriano F., *The Inviscid Limit for the Navier–Stokes Equations with Slip Condition on Permeable Walls*, Journal of Nonlinear Science. **23**, n.5, 731–750, (2013).

Hard collisions of elastic bodies in solid mechanics

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Solid/solid collisions are commonly modelled using a phenomenological approach, e.g. by adding an artificial soft repulsion potential. While this is often beneficial from the point of calculations, it turns out to not be strictly necessary. Instead, for bulk elastic solids, the physics can be fully determined by only prescribing the non-interpenetration of matter. The resulting problem however is highly non-convex. Nevertheless, we are able to show long-time existence of solutions and can derive some of their properties.

Image binarization method for the detection of markers in bad lighting conditions

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3D scanning technology has become indispensable in many fields of science and industry. Structured light scanning systems use markers for coupling multiple scans into one and for the calibration process. Under extreme conditions such as light reflection or non-uniform illumination, markers detection can be a very demanding task. Incorrect marker positions result in incorrect calibration parameters, i.e. incorrect mathematical models, that result in distorted and incorrectly scaled scans of the desired object.

We present an image binarization method that is primarily used to find markers such as those used in mobile 3D scanning systems. Handling a mobile 3D scanning system often includes bad lighting conditions. The proposed binarization method, as the fundamental part of the scanning process, successfully overcomes the above problems. Due to the trend of increasing image size and real-time image processing, we were able to achieve the required low algorithmic complexity.

Solving polynomial and rational eigenvalue problems through linearizations

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In numerous applications, such as vibration analysis of machinery, buildings, and vehicles, control theory and linear systems theory, and approximation of various nonlinear eigenvalue issues, polynomial and rational matrices are used. We will explore how to develop a new family of Fiedler-like linearizations (named EGFPR) from the given data of the eigenvalue problems in order to provide a direct solution for polynomial and rational eigenvalue problems. EGFPR broadens the class of linearizations by incorporating all Fiedler-like pencils in the literature, resulting in a huge number of structure-preserving linearizations for structured (Symmetric, Hermitian, Palindromic, and other) eigenvalue problems. Finally, we discuss operation-free eigenvector and minimal base recovery, as well as minimal indices of polynomial and rational matrices.

References:

[1] R. K. Das and R. Alam, Palindromic linearizations of palindromic matrix polynomials of odd degree obtained from Fiedler-Like pencils, Vietnam Journal of Mathematics, 48, pp 865–891, 2020.

[2] R. K. Das and R. Alam, Structured strong linearizations of structured rational matrices, Linear and Multilinear Algebra, pp 1-34 , 2021.

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On a numerical method for data driven nonlinear system identification

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We discuss recently proposed method for data driven identification of nonlinear systems, that is based on lifting the data snapshots into a suitable finite dimensional function space and identification of the infinitesimal generator of the Koopman semigroup. Using two different representations of a compression of the generator, one can reveal the structure of the underlying dynamics (that generated the data). This elegant and mathematically appealing approach has good analytical (convergence) properties, but numerical experiments show that software implementation of the method has certain limitations. In this talk, we will present numerical aspects/difficulties of the computer implementation of the method. It will be shown how the state of the art numerical linear algebra can be used to improve the numerical performances in the cases that are usually considered notoriously ill-conditioned. We use a preconditioning technique for more accurate computation of the matrix logarithm, as well as a variant of dictionary pruning scheme to make the computation with a compression of the Koopman operator (and its logarithm) numerically more stable. Numerical experiments show that the proposed techniques greatly improve the robustness of the method.

Strong traces of entropy solutions to degenerate parabolic equations on the boundary

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In this talk we study entropy solutions of degenerate (homogeneous) scalar parabolic equations on a bounded domain and prove that, under certain conditions, all such solutions admit the strong trace at the boundary. Here the degeneracy appears as the diffusion matrix is only positive semi-definite, i.e. it can be equal to zero in (variable-dependent) directions. Although the well-posedness for the Cauchy problems (under the strong trace) corresponding to such equations is well established in quite general situations, our result implies that the weak trace suffices for uniqueness. Moreover, this result could be an important step into formulating the initial boundary value problem in the sense of Bardos, LeRoux and Nédélec.

Regularity of a weak solution to a linear fluid-composite structure interaction problem

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In this work, we deal with the regularity of a weak solution to the fluid-composite structure interaction problem which describes a linear fluid-structure interaction between an incompressible, viscous fluid flow, and an elastic structure composed of a cylindrical shell supported by a mesh-like elastic structure. The fluid and the mesh-supported structure are coupled via the kinematic and dynamic boundary coupling conditions describing continuity of velocity and balance of contact forces at the fluid-structure interface. Recently it was shown that there exists a weak solution to the described problem. By using the standard techniques from the analysis of partial differential equations we prove that such a weak solution possesses an additional regularity in both time and space variables for initial and boundary data satisfying the appropriate regularity and compatibility conditions imposed on the interface.

A data-driven reformulation of a hybrid projection-based model reduction method

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Model order reduction (MOR) techniques are of particular importance for approximating large-scale, complex models of dynamical systems, with much simpler and smaller dimensional surrogate models that can be easily simulated, controlled or further analyzed. A generic goal for most MOR methods, is to provide reduced-order models (ROMs) which are close, in terms of response, characteristics, properties, to the original complex, potentially large-scale model. An important category of MOR, particularly relevant for this study, is the subclass of data-driven non-intrusive methods, for which access to the original operators characterizing the dynamics is not necessary; in these cases, only (measured) data is needed.

We propose a data-driven, realization-free reformulation of the iterative SVD-Krylov algorithm (ISRK), projection-based (two-sided) model reduction method originally proposed in [1]. Our reformulation of ISRK has been inspired by the data-driven quadrature-based approximate balanced truncation method developed recently in [2]. The original formulation of ISRK is based on a two-sided projection and combines rational Krylov-based and balancing (SVD) techniques. The SVD component of ISRK depends on the infinite observability Gramian, while the Krylov-side is obtained via iterative steps performed as in the iterative rational Krylov algorithm (IRKA) introduced in [3]. It was shown in [1] that the reduced model thus obtained is indeed asymptotically stable, matches certain moments corresponding to the original system, and solves a restricted $\mathcal{H}_2$ minimization problem.

In this newly-developed data-driven approach, we show that ISRK steps do not necessarily require access to the original system's matrices, but only to data. In this context, data correspond to samples of the system's transfer function (TF), evaluated at particular values. Basically, we circumvent the need for explicitly computing the observability Gramian, and interpret all necessary matrices as data quantities (that can be computed in terms of appropriately-sampled TF values). We test the new approach for various MOR benchmarks, and compare its performance to ISRK and IRKA.

References:

[1] S. Gugercin. An iterative SVD-Krylov based method for model reduction of large-scale dynamical systems. Linear Algebra and its Applications, 428(8-9):1964–1986, 2008.

[2] I. V. Gosea, S. Gugercin, C. Beattie. Data-driven balancing of linear dynamical systems, SIAM Journal on Scientific Computing 44 (1), A554-A582, 2022.

[3] S. Gugercin, A.C. Antoulas, and C. Beattie. $\mathcal{H}_2$ model reduction for large-scale linear dynamical systems, SIAM Journal on Matrix Analysis and Applications, 30(2):609–638, 2008.

Asymptotic behaviour of solutions to the wave equation with variable coefficients

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We investigate the asymptotic behaviour of the wave equation on $\langle 0, T\rangle \times \Omega$

$$\begin{cases} (\rho_n(x,t)u_n')' - \operatorname{div}(A_n(x,t)\nabla u_n) = f_n, & \\ u_n = 0, & \text{on } \langle 0, T\rangle \times \Gamma, \\ u_n(0,\cdot) = u_n^0, & \\ (\rho_n u_n')(0,\cdot) = u_n^1, & \end{cases}$$

where the coefficients are in $BV(0,T;L^\infty(\Omega))$.

The problem exhibits two different comportments, one where the oscillation in the coefficients is dominant, therefore leading to the homogenisation problem, and the other, where the propagation of oscillations generated by initial conditions is dominant, which can be approached by using H-measures.

Surrogate Modelling for Detection of Near Resonances in Acoustic Scattering

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We study real quasi resonant modes (approximate eigenpairs) of a non self-adjoint spectral problem in acoustic scattering. For this class of problems one can prove that the relevant resolvent has only complex poles (eigenvalues), which cluster close to the real axis. A consequence of this clustering is the emergence of many real "almost" eigenvalue eigenvector pairs with the small residue. These modes correspond to the so called whispering gallery modes. These modes are strong (in the energy norm sense) fields localised near the boundary of the domain and their rigorous numerical analysis is the focus of this study. We will present details of the algorithm together with a validating numerical case study.

Subspace acceleration method for simulation of plasma in tokamak boundary

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Large scale simulations of the plasma boundary are key in the developments of tokamak fusion experiments. A collaboration with Swiss Plasma Center institute led to the development of a new method in order to accelerate the solution of consecutive linear systems that can arise from such simulations. In particular, a system of differential algebraic equations of first order is obtained from the three-dimensional drift-reduced Brangiskii equations that model plasma dynamics in the edge layer of the tokamak. The explicit integration in time of the obtained system result in the need of solving a sequence of consecutive linear systems where both the matrix and the right-hand-side are time dependent, and the time-step is constrained to be small from the explicit nature of the integration. The idea is to leverage the history of previous time solutions to choose an initial guess for the iterative solver that is employed in the resolution of the linear system, drastically reducing the number of iterations needed, hence the computational time. Inspired by consideration already existing for implicitly discretized nonlinear evolution problems, the initial guess can be obtained from the solution a reduced order model, that can be constructed using different projection techniques. The new method is presented in a theoretical framework, analyzing how the smooth dependence of the state variables from time can affect the approximation, while its effectiveness is proved showing the speed-up achieved in large scale plasma simulations.

This talk is based on joint work with Daniel Kressner and Paolo Ricci.

Randomized Joint Orthogonal Diagonalization of Real Symmetric Matrices

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The classical problem of joint diagonalization of an almost commuting family of real symmetric matrices is to find an orthogonal matrix that almost diagonalizes every matrix in the family. Most current methods solve this problem using optimization techniques. We propose a novel randomized method that reduces the problem to a standard eigenvalue problem through random linear combinations. Unlike existing optimization-based methods, our algorithm is easy to implement and leverages existing high-quality linear algebra software packages. We prove its robustness by proving that the backward error of the output orthogonal joint diagonalizer will be small with high probability through a perturbation analysis. The algorithm can

be further improved by deflation techniques. Through numerical experiments on synthetic and real-world data, we show that our algorithm reaches state-of-the-art performance.

Two-phase optimal design for elastic plate

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Homogenization theory is one of the most successful approaches for dealing with optimal design problems, that consists in arranging given materials such that obtained body satisfies some optimality criteria, which is mathematically usually expressed as minimization of some (integral) functional under some (PDE) constrains.

We consider optimal design problems in the setting of the Kirchhoff-Love equation describing an elastic, thin, symmetric plate, which is a fourth order elliptic equation, and we restrict ourselves to domains filled with two isotropic elastic materials. Since the classical solution usually does not exist, we use relaxation by the homogenization method in order to get a proper relaxation of the original problem. Unfortunately, the set of all possible homogenized elastic materials is not known (the famous G-closure problem). However, for the compliance functional, one can overcome this by knowledge of the Hashin-Shtrikman bounds. More precisely, the necessary conditions of optimality are easily derived and expressed via lower Hashin-Shtrinkman bound on the complementary energy, which enables a development of the optimality criteria method for finding an approximate solution.

Moreover, we consider optimal design problems in the small-amplitude regime when the two materials being employed are not very different, i.e. the quotient of their relevant physical properties is close to 1. This approach has application in the context of inverse problems when the defect inside a media is caused, e.g., by weak inhomogeneities during the fabrication process.

Existence of a weak solution to a compressible two-component fluid-structure interaction problem

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The talk is devoted to the analysis of a system of PDEs governing the interaction between two compressible mutually noninteracting fluids and a shell of Koiter type encompassing a time dependent domain filled by the fluids. The system of equations consists of the three–dimensional momentum equation with the pressure possessing an involved structure dependent on two variable densities, continuity equations for the density of each component and the equation for a displacement of the boundary of the physical domain with the gradient of a linearized version of the Koiter elastic energy. We prove the existence of a weak solution to the system until the time–dependent boundary approaches a self–intersection applying techniques of decoupling, penalization and domain extension.

Numerical methods for the segmentation of Natura 2000 habitats in Sentinel-2 satellite images

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The contribution presents an overview of mathematical models based on solving partial differential equations on evolving curves, and numerical methods based on finite volume method, designed for the segmentation of Natura 2000 habitats in Sentinel-2 satellite images. These methods are implemented in the NaturaSat software which allows botanists, environmentalists and nature conservationists across Europe to explore protected areas of Natura 2000 habitats using the Sentinel-2 optical data. Sentinel-2 Mission is an Earth observation mission that systematically acquires optical multi-spectral data at a high resolution from two separate Sentinel-2 satellites. The presented methods are designed for accurate area identification - semi-automatic and automatic segmentation of European protected habitats and monitoring of their spatio-temporal distribution and quality.

The first implemented segmentation tool is semi-automatic segmentation which allows a user to manually segment areas using a mouse cursor. By clicking, users create a nonlinear open curve, which is driven by constructed vector field towards the boundary of the segmented area. As the second segmentation tool, we use the automatic segmentation model, where we consider the time evolution of a closed planar segmentation curve driven by the velocity defined as a combination of the

movement in the normal direction and tangential direction. Movement in the normal direction represents a combination of three functions with different behaviour. The first function is an expanding term. This function drives the segmentation curve from its initial position through the segmented area to its border. The second function is the edge attracting term. This function attracts the evolving curve accurately to the boundary of the segmented area. The last function represents the curvature term which regularizes and smooths the shape of the evolving curve.

These methods are verified by experts from Plant Science and Biodiversity Centre of the Slovak Academy of Sciences and State Nature Conservancy of the Slovak Republic and applied in different locations across Europe along the Danube river alluvium. At the end of the contribution, we also present examples of results from these locations.

Improved parallel-in-time integration via low-rank updates and interpolation

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This talk is concerned with linear matrix equations that arise from the space-time discretization of time-dependent linear partial differential equations (PDEs). Such matrix equations have been considered, for example, in the context of parallel-in-time integration leading to a class of algorithms called ParaDiag. We develop and analyze two novel approaches for the numerical solution of such equations. Our first approach is based on the observation that the modification of these equations performed by ParaDiag in order to solve them in parallel has low rank. Building upon previous work on low-rank updates of matrix equations, this allows us to make use of tensorized Krylov subspace methods to account for the modification. Our second approach is based on interpolating the solution of the matrix equation from the solutions of several modifications. Both approaches avoid the use of iterative refinement needed by ParaDiag and related space-time approaches in order to attain good accuracy. In turn, our new approaches have the potential to outperform, sometimes significantly, existing approaches. This potential is demonstrated for several different types of PDEs.

A stochastic fluid-structure interaction problem describing Stokes flow interacting with a membrane

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In this talk, we present a well-posedness result for a stochastic fluid-structure interaction model. We study a fully coupled stochastic fluid-structure interaction problem, with linear coupling between Stokes flow and an elastic structure modeled by the wave equation, and stochastic noise in time acting on the structure. Such stochasticity is of interest in applications of fluid-structure interaction, in which there is random noise present which may affect the dynamics and statistics of the full system. We construct a solution by using a new splitting method for stochastic fluid-structure interaction, and probabilistic methods. To the best of our knowledge, this is the first result on well-posedness for fully coupled stochastic fluid-structure interaction.

Several regularity criteria for solutions of the Navier-Stokes equations

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We study the conditional regularity of the weak solutions of the Navier-Stokes equations for incompressible fluid on a bounded domain with Navier's boundary conditions, Navier-type boundary conditions or Dirichlet boundary conditions. We prove some regularity criteria based on the directional derivatives of several fundamental variables along the streamlines, namely the velocity magnitude, the kinetic energy, the pressure and the velocity field. These criteria are optimal ones and of clear physical meaning.

Numerical validation of fluid-poroelastic structure interaction solvers on benchmark examples

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We are interested in a multiphysics interaction problem of the fluid and the deformable poroelastic structure. For the fluid problem we consider a viscous, incompressible Newtonian fluid governed by the Stokes equations. The poroelastic structure is described by the Biot model in which the fluid movement and solid skeleton deformation occur simultaneously. On the interface interaction between the fluid and the poroelastic structure are given with the coupling conditions.

Several different monolithic schemes are presented. They differ in the numerical treatment of the coupling conditions, i.e. the presence of the Nitsche's method for the enforcement of the normal velocity continuity condition.

Two standard benchmark examples were used to validate presented monolithic schemes. The first example uses the exact solutions to test the accuracy of the schemes. The second example follows the propagation of a single pressure wave whose amplitude is comparable to the pressure difference between systolic and diastolic phases of the heartbeat. In the example with the exact solutions, all methods offer similar results without noticeable differences in the rate of convergence with mesh and time step refinement. In the latter example, only the monolithic schemes with the Nitsche's mortaring offers stable approximations of the solutions which do not change with the finer mesh or the smaller time step size.

Optimal design of a shell with respect to its thickness

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In this work the mathematical theory and computational algorithm to study optimal design of shells by varying the shell's thickness in order to minimize the overall compliance will be presented. The shell model used is of the Naghdi type while the minimization algorithm is of the gradient type. Simulations using self-implemented finite elements in Matlab will also be presented. The problem of finding optimal thicknesses of triangles in the shell's mesh, keeping the overall volume of the shell fixed, without and with dimension reduction using proper orthogonal decomposition will be presented.

RANDOMIZED DYNAMICAL LOW-RANK APPROXIMATION

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A randomized scheme for the time-evolution of the low-rank approximation of matrix differential equations is presented. The scheme constructs the time-dependent approximation of the range and co-range of the solution via random sketches and obtains the low-rank approximation by the generalized Nyström's method. It is shown that the scheme retains an exactness property in expectation. Under suitable approximability assumptions, a robust error bound is expected to hold in expectation. Numerical experiments illustrating the properties of the proposed randomized scheme are presented. This is ongoing joint work with Daniel Kressner and Gianluca Ceruti.

Constrained LQ optimization problems with parameter dependent coefficients

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We consider a sequence of linear-quadratic optimization problems defined in an abstract functional framework. Each problem is accompanied by the constraint of reaching a given target with some precision. We show that problems are well posed and we characterise their solutions. The main result provides the conditions under which these solutions converge to the minimizer of the limit problem.

The theory can be applied to a wide range of problems: elliptic, parabolic, control ones etc. In the talk, the following examples will be presented:

1. optimal controllability of the heat equation,

2. optimal solvability of the Poisson equation.

Modelling interaction between 3d structure, 2d membrane plate and 1d rod

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In this talk we rigorously derive models for interaction of three possibly different threedimensional (3d) linearized elastic objects: a cube, a layer of thickness ε attached to the cube and a rod of thickness and width equal to ε inside that layer. It is assumed that the elasticity coefficients for the layer are of order $1/\varepsilon$, and that for rod are of order $1/\varepsilon^q$, $q > 0$. Derivation is observed when parameter ε tends to 0. In the limit five different models are obtained with respect to different choices of q. Moreover, a 3d-2d-1d model is proposed which has the same asymptotics as the original three-dimensional problem. This is convenient for applications since one do not have to decide in advance which limit model to use. This is a joint work with J. Tambača.

A mixed precision Jacobi algorithm

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We propose a mixed precision Jacobi algorithm for computing the singular value decomposition (SVD) of a dense matrix. After appropriate preconditioning, the proposed algorithm computes the SVD in a lower precision as an initial guess, and then performs one-sided Jacobi rotations in the working precision as iterative refinement. By carefully transforming a lower precision solution to a higher precision one, our algorithm achieves about 2× speedup on the x86 architecture compared to the usual one-sided Jacobi SVD algorithm in LAPACK, without sacrificing the accuracy.

Analysis of the adiabatic piston problem via methods of continuum mechanics

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We consider a system modelling the motion of a piston in a cylinder filled by a viscous heat conducting gas. The piston is moving longitudinally without friction under the influence of the forces exerted by the gas. In addition, the piston is supposed to be thermally insulating (adiabatic piston). This fact raises several challenges which received a considerable attention, essentially in the statistical physics literature. We study the problem via the methods of continuum mechanics, specifically, the motion of the gas is described by means of the Navier-Stokes-Fourier system in one space dimension, coupled with Newton's second law governing the motion of the piston. We establish global in time existence of strong solutions and show that the system stabilizes to an equilibrium state for $t \to \infty$.

Boundary condition on porous boundary

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We derive the new effective boundary condition for the fluid flow in domain with porous boundary. Starting from the Newtonian fluid flow through a domain with an array of small holes on the boundary, using the homogenization and the boundary layers, we find an effective law in the form of generalized Darcy law. If the pores geometry is isotropic, then the condition splits in Beavers-Joseph type condition for the tangential flow and the standard Darcy condition for the normal flow.

The influence of solver tolerances on large scale coupled climate simulations

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Subject of the talk is the influence of numerical tolerances as well the development of a new Newton solver for a specific problem in climate science, the Snowball Earth hypothesis: 630-750 million years ago the Earth might have been in deep freeze with completely ice-covered oceans. Climate simulations that investigate the Snowball Earth hypothesis typically approximate the nonlinear sea ice processes with only a few solver iterations due to the high numerical costs. The influence of this inaccuracy as well as the development of fast and robust solving methods are current research questions.

We show that the underlying coupled set of equations that describes the sea ice dynamics in climate models can be formulated as an energy minimization problem. Based on the theoretical analysis we derive a new Newton method that leads to faster and more robust Newton convergence than currently used methods. In the context of the Snowball Earth hypothesis, we demonstrate that the numerical tolerances are more important for the resulting climate dynamics than usual tuning parameters. These are alarming results, since the numerical accuracy of nonlinear sea ice processes have never been considered to be important for past, present, or future climate projections.

Iterative rational Krylov algorithm and Riemannian optimization

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Iterative rational Krylov algorithm (IRKA) is a method for $\mathcal{H}_2$-optimal model order reduction. The $\mathcal{H}_2$-optimal model order reduction problem is given as

$$\underset{\hat{H}\in\mathcal{M}}{\text{minimize}} \quad \|H-\hat{H}\|_{\mathcal{H}_2},$$

where H is an element of the Hardy space $\mathcal{H}_2^{p\times m}$, which is an infinite-dimensional Hilbert space with norm $\|\cdot\|_{\mathcal{H}_2}$. The set $\mathcal{M}\subset\mathcal{H}_2^{p\times m}$ consists of reduced-order models of fixed order. We show that the set $\mathcal{M}$ can be given the structure of a Riemannian embedded submanifold of $\mathcal{H}_2^{p\times m}$. Based on this, we find that IRKA can be viewed as a Riemannian gradient descent method with a fixed step size. This motivates developing other Riemannian optimization methods in this setting. We demonstrate these theoretical results on a few numerical examples.

On the problem of compressible Navier–Stokes system with the hard sphere pressure law

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We consider the motion of compressible Navier–Stokes fluids with the hard sphere pressure law around a rigid obstacle when the velocity and the density at infinity are non zero. This kind of pressure model is largely employed in various physical and industrial applications. We prove the existence of weak solutions to the system in the exterior domain. It is a joint work with A. Novotny and A. Roy.

Numerical analysis and computing of a one-dimensional moving boundary problem capturing the penetration of diffusants into rubbers

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We propose a moving-boundary scenario to model the penetration of diffusants into rubbers. Immobilizing the moving boundary by using the well known Landau transformation transforms the original governing equations into new equations posed in a fixed domain. We solve the transformed equations by the finite element method. Numerical simulation results show that the computed penetration depths of the diffusant concentration are within the range of experimental measurements. To have trust in the obtained simulation results, we perform the numerical analysis for our setting. Initially, we study semi-discrete finite element approximations of the corresponding weak solutions. We prove both *a priori* and *a posteriori* error estimates for the mass concentration of the diffusants, and respectively, for the *a priori unknown* position of the moving boundary. Finally, we present a fully discrete scheme for the numerical approximation of model equations. Our scheme is based on the Galerkin finite element method for the space discretization combined with the backward Euler method for time discretization. In addition to proving the existence and uniqueness of a solution to the fully discrete problem, we also derive *a priori* error estimates for the mass concentration of the diffusants, and respectively, for the position of the moving boundary. Our numerical illustrations verify the obtained theoretical order of convergence in physical parameter regimes.

Multi-Scale Modeling and Simulation of Transport Processes in an Elastically Deformable Perforated Medium

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In this paper, we derive an effective model for transport processes in periodically perforated elastic media, taking into account, e.g., cyclic elastic deformations as they occur in lung tissue due to respiratory movement. The underlying microscopic problem couples the deformation of the domain with a diffusion process within a mixed Lagrangian/Eulerian formulation. After a transformation of the diffusion problem onto the fixed domain, we use the formal method of two-scale asymptotic expansion to derive the upscaled model, which is nonlinearly coupled through effective coefficients. The effective model is implemented and validated using an application-inspired model problem. Numerical solutions for both, cell problems and macroscopic equations, are investigated and interpreted. We use simulations to qualitatively determine the effect of the deformation on the transport process. This research is supported by SCIDATOS (Scientific Computing for Improved Detection and Therapy of Sepsis), a collaborative project funded by the Klaus Tschira Foundation, Germany.

The Natural Numerical Network as a new classification method

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The presented study aims to introduce a new deep learning algorithm for classification and one of its possible uses in nature conservation practices. The natural numerical network represents an algorithm of supervised deep learning, and it is based on the numerical model of nonlinear forward-backward diffusion on the complete undirected graph. With a suitable combination of forward and backward diffusion, we achieve the classification effect without calculation instabilities. Forward diffusion, used in the model, causes moving points to each other in the graph, thus clustering the points together. The opposite effect, the repulsion of the points belonging to different clusters, is caused by backward diffusion. The vertices in the graph

are equivalent to areas around chosen pixels on the satellite image. The characteristic features are calculated for these areas from satellite optical bands, and they represent the dimension of the vertices. Moreover, the information about elevation level is added to the features space. The obtained feature space is multidimensional; thus, reducing the dimension is necessary, and Principal Component Analysis provides it. The natural numerical network is a desirable tool for classifying Natura 2000 habitats. The Natural 2000 represents a complex plant community with nontrivial character, which is difficult to classify, and it is an open environmental and nature conservation problem. The introduced algorithm can automatically identify habitats using satellite image information. The process of identification and classification is divided into two phases. First is the learning phase, where the training of the network is done, which means that the parameters of the natural numerical network and the network graph topology are optimized. The second is the validation phase, in which the segmented areas are classified, and the relevancy of the classification is calculated. Based on the calculation of the relevancy, we can create the relevancy map, which gives us information about the relevancy of each pixel in the satellite image. The relevancy maps are used for identifying new appearances of the Natura 2000 habitats.

Trace maximization algorithm for the approximate tensor diagonalization

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In this talk we introduce a Jacobi-type algorithm for the approximate diagonalization of tensors of order $d \geq 3$ via tensor trace maximization. For a general tensor this is an alternating least squares algorithm and the rotation matrices are chosen in each mode one-by-one to maximize the tensor trace.

Unlike matrices, not every symmetric tensor can be diagonalized using orthogonal transformations. For symmetric tensors we discuss a structure-preserving variant of our algorithm where in each iteration the same rotation is applied in all modes.

Both versions of the algorithm converge to the stationary points of the corresponding objective functions. We present convergence theorems and several numerical examples.

Low rank approximations of matrix functions

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Randomized low-rank approximation, such as the randomized singular value decomposition and the randomized Nyström approximation, has lately gained increased attention as it provides a cheaper, yet accurate, alternative to deterministic methods. In some applications, one seeks to obtain a low-rank approximation of a matrix function $f(A)$, where A is symmetric positive semi-definite. Often, $f(A)$ cannot be accessed explicitly and not even implicitly via exact matrix-vector products, but $f(A)$ can only be accessed via *approximate* matrix-vector products. Moreover, in many cases, computing Ax is significantly cheaper than computing an approximation to $f(A)x$. Consequently, obtaining a low-rank approximation of A can be substantially cheaper than obtaining a low-rank approximation of $f(A)$ using standard randomized methods. In this work, we present a method to construct a low-rank approximation of $f(A)$ for monotone functions f satisfying $f(0) = 0$ and symmetric positive semi-definite A. Our method utilizes the Nyström approximation of A and requires no approximations to matrix-vector products with $f(A)$. For the special case of an operator monotone f we provide strong probabilistic error bounds. Numerical experiments demonstrate the effectiveness of our method.

Finite time horizon mixed control of vibrational systems

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We consider a vibrational system control problem over a finite time horizon. The performance measure of the system is taken to be p-mixed H_2 norm which generalizes the standard H_2 norm. We present an algorithm for efficient calculation of this norm in the case when the system is parameter dependent and the number of inputs or outputs of the system is significantly smaller than the order of the system. Our approach is based on a novel procedure which is not based on solving Lyapunov equations and which takes into account the structure of the system. We use a characterization of the H_2 norm given in terms of integrals which we solve using adaptive quadrature rules. This enables us to use recycling strategies as well as parallelization. The efficiency of the new algorithm allows for an analysis of the influence of various system parameters and different finite time horizons on the value of the p-mixed H_2 norm. We illustrate our approach by numerical examples concerning an n-mass oscillator with one damper.

Models of compressible chemically reacting mixtures with Dirichlet boundary conditions for the temperature

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Based on recent results for the steady compressible Navier–Stokes–Fourier system with Dirichlet boundary conditions for the temperature (see [1] or [2]) we reconsider the steady models of chemically reacting heat conducting mixtures (see [3], [4] or [5]) and study the problem of existence of solutions for the system of partial differential equations in a bounded sufficiently smooth set $\Omega \subset R^3$

$$\begin{aligned} \operatorname{div}(\varrho \mathbf{u}) &= 0, \\ \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) - \operatorname{div} \mathbb{S} + \nabla \pi &= \varrho \mathbf{f}, \\ \operatorname{div}(\varrho E \mathbf{u}) + \operatorname{div}(\pi \mathbf{u}) + \operatorname{div} \mathbf{Q} - \operatorname{div}(\mathbb{S}\mathbf{u}) &= \varrho \mathbf{f} \cdot \mathbf{u}, \\ \operatorname{div}(\varrho Y_k \mathbf{u}) + \operatorname{div} \mathbf{F}_k &= m_k \omega_k, \quad k \in \{1, \dots, n\} \end{aligned} \tag{1}$$

with either the homogeneous Dirichlet or Navier boundary conditions for the velocity on $\partial\Omega$, and

$$\begin{aligned} \mathbf{F}_k \cdot \mathbf{n} = 0, \quad k = 1, 2, \dots, n, \\ \vartheta = \vartheta_D. \end{aligned} \tag{2}$$

In dependence on the parameters $\gamma > 1$ (the exponent describes the growth of pressure for large total density) and m (the exponent describes the growth of the heat conductivity for large temperature) we show existence of variational entropy or weak solutions to this problem without any restriction on the size of the data.

References:

[1] M. Pokorný: *Steady Compressible Navier–Stokes–Fourier Equations with Dirichlet Boundary Condition for the Temperature.* J. Math. Fluid Mech. **24** (2022), no. 1, 17.

[2] T. Piasecki, M. Pokorný: *Steady Compressible Navier–Stokes–Fourier Equations with Navier boundary conditions for the velocity and Dirichlet Boundary Condition for the Temperature.* In preparation.

[3] T. Piasecki, M. Pokorný: *On steady solutions to a model of chemically reacting heat conducting compressible mixture with slip boundary conditions.* Mathematical analysis in fluid mechanics—selected recent results, 223–242, Contemp. Math., **710**, Amer. Math. Soc., [Providence], RI, 2018.

[4] T. Piasecki, M. Pokorný: *Weak and variational entropy solutions to the system describing steady flow of a compressible reactive mixture.* Nonlinear Anal. **159** (2017), 365–392.

[5] V. Giovangigli, M. Pokorný, E. Zatorska: *On the steady flow of reactive gaseous mixture.* Analysis (Berlin) **35** (2015), no. 4, 319–341.

Derivation of a model describing the fluid flow in a reservoir with small holes

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We study the flow of the fluid in a reservoir with several small holes. The velocities on holes (injection and ejection) are prescribed and high. As the size of each holes shrinks to one point, the boundary value degenerates to a Dirac masses concentrated on the boundary. We prove that the solution of the Stokes system, describing the original flow, converges to the very-weak solution of the Stokes system with Dirac measures on the boundary.

Model reduction of descriptor systems with quadratic output equations

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This work is dedicated to the model reduction of differential-algebraic systems with quadratic output (DAE_QO) equations of the form

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \qquad x(0) = 0, \\ y(t) &= x(t)^T M x(t). \end{aligned}$$

Under some mild conditions, these systems can be transformed into a Weierstraß canonical form and thus decouple into a differential equation and an algebraic equation and the corresponding differential and algebraic states. In contrast to the case of differential-algebraic systems with linear output (DAE_LO), the system DAE_QO presents a coupling of the differential and algebraic states in the output signal. Our goal is to determine the dominant subspaces of the differential and algebraic states and reduce the system accordingly.

To this end, we propose new Gramians that encode the suitable subspaces and we demonstrate their relationship with energy functions. Firstly, we show that reachability is encoded by the proper and improper reachability Gramians, similarly to the DAE_LO case. For the observability, we decompose the system output into four subsystems, two of those associated with differential states and the other two

with algebraic states. Based on this subsystem decomposition, we derive new observability Gramians for the differential and algebraic state vectors. They can be determined by solving continuous-time and discrete-time Lyapunov equations. Finally, the proper reachability and observability Gramians are then used to derive a reduced differential state space using balanced truncation. Additionally, the improper Gramians are used to truncate uncontrollable and unobservable algebraic states. Furthermore, we derive an error estimator which is used to evaluate the quality of the reduced surrogate model. Finally, we illustrate the effectiveness of our method by applying it to example problems.

Rigid body in compressible flow with general inflow-outflow boundary data

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We study the motion of a rigid body in a compressible isentropic viscous fluid contained in a fixed bounded domain $\Omega \subset \mathbb{R}^3$. The equations of motion of the fluid are the Navier-Stokes equations and the equations for the body are ordinary differential equations describing the conservation of linear and angular momentum. We prescribe a time-independent fluid velocity at the boundary Ω and time-independent fluid density at the inflow boundary of Ω, while at the boundary of the rigid body we assume no-slip boundary condition. Our aim is to prove existence of global-in-time weak solution for this problem provided that the rigid body does not touch the boundary of Ω. This is a joint work with Šimon Axmann and Šárka Nečasová.

Homogenization and dimension reduction of reaction diffusion problem with polynomial type drift

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We study the question of periodic homogenization of a variably scaled reaction-diffusion equation with non-linear drift of polynomial type. The non-linear drift was derived as the hydrodynamic limit of a totally asymmetric simple exclusion process (TASEP) for a population of interacting particles crossing a domain with obstacles. We consider different geometries: (i) Bounded domain crossed by a finitely thin flat composite layer; (ii) Bounded domain crossed by an infinitely thin flat composite layer; For the thin layer cases, we consider our reaction-diffusion problem endowed with slow or moderate drift. Using energy-type estimates as well as concepts like

thin-layer convergence and two-scale convergence, we derive homogenized evolution equations and the corresponding effective model parameters. Special attention is paid to the derivation of the effective transmission conditions across the separating limit interfaces. This study aims to contribute to the theoretical understanding needed when designing thin composite materials that are resistant to slow, moderate, and high velocity particle impacts. This is work done in collaboration with Adrian Muntean (Karlstad, Sweden), Ida de Bonis(Rome, Italy) and Emilio N. M. Cirillo (Rome, Italy).

Numerical simulation of vibration induced rigid body motion

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We study the motion of a rigid body that is immersed in a fluid and exposed to a periodic force with average zero. We investigate the fundamental question of whether it is possible to give the body a non-zero average velocity even though the acting forces add up to zero. The motivation to study this problem numerically is an ongoing collaboration with Paolo Galdi, who approaches the problem from the theoretical side. In this talk, numerical methods for efficient simulation are described and various parameter studies are presented.

An essential problem is that a periodically acting force can lead to an oscillating motion with large amplitude, but that it is hardly possible to decide whether the average velocity is really different from zero. We discuss acceleration possibilities for identifying true periodic solutions.

Learning fixed points in dynamical recurrent neural network models

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Recurrent neural network models are widely used in machine learning for problems involving time series. Specifically, they can learn mappings from input time series to output time series. In contrast, recurrent neural network models in computational neuroscience are dynamical systems for which the fixed points are typically the relevant quantities of interest. In this study, we derive learning rules for training the weights of dynamical recurrent neural network models to learn mappings from static inputs to static fixed points. We find that the standard approach using gradient descent on a loss surface with respect to the weights performs poorly for learning

fixed points. We reparameterize the dynamical model to derive a new learning rule. A linearization of the learning rule derived from this reparameterized model is simple, computationally efficient, and produces effective learning dynamics. We show that this new rule is equivalent to gradient descent with a non-Euclidean metric on the parameter space.

Asymptotic analysis of the thermomicropolar fluid flow through a curvilinear channel

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In this talk, we consider the steady flow of a thermomicropolar fluid through a thin curvilinear channel. The flow is governed by the prescribed pressure drop between the channel's ends. The heat exchange between the fluid inside the channel and the exterior medium is allowed through the upper curved wall, while the lower wall is insulated. Using the technique of two–scale expansion, we compute the asymptotic approximation of the solution. The solution is obtained in explicit form, capturing the effects of the curvature of the domain as well as the micropolarity of the fluid. A boundary layer analysis is provided for the microrotation. The proposed effective model is rigorously justified by proving the error estimates in suitable norms.

Time-periodic solutions for fluid-structure interactions

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Under the action of a time-periodic external force we prove the existence of at least one time-periodic weak solution for the interaction between a three-dimensional incompressible fluid, governed by the Navier–Stokes equation and a two-dimensional elastic plate. The challenge is that the Eulerian domain for the fluid changes in time and is a part of the solution. We introduce a two fixed-point methodology. First we construct a time-periodic solutions for a given variable time-periodic geometry. Then in a second step a fixed point is performed w.r.t. the geometry of the domain. The existence relies on newly developed a priori estimates applicable for both coupled and uncoupled variable geometries. Due to the expected weak regularity of the solutions such Eulerian estimates are unavoidable. Note in particular that only the fluid is assumed to be dissipative, while the solid is hyperbolic.

Hermite interpolation on manifolds with retractions

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Interpolation of data belonging to non-Euclidean spaces is an active research area fostered by its numerous applications. In this work we focus on the Hermite interpolation problem: finding a smooth manifold curve that interpolates a collection of data points belonging to a Riemannian manifold while matching prescribed velocities at each point. We propose a novel procedure relying only on retractions to solve this problem on a large class of manifolds, including those for which tractable computations of the Riemannian exponential and logarithm maps are not available, like for instance the manifold of fixed rank matrices. We analyze the well-posedness of the method by introducing and showing the existence of retraction convex sets, generalization of geodesically convex sets. We extend to the manifold setting the classical result characterizing the asymptotic interpolation error of Hermite interpolation for the case where data points are equispaced samples from an underlying smooth manifold curve. We finally illustrate these results and the effectiveness of the method with numerical experiments on the manifold of fixed rank matrices and the Stiefel manifold of column orthogonal matrices.

Convergence analysis of a MAC scheme for the compressible Navier-Stokes system

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We study the unconditional convergence of a Marker-And-Cell scheme for the compressible Navier-Stokes system via the concept of dissipative weak-strong uniqueness principle for physically relevant adiabatic coefficient $\gamma > 1$. Moreover, we estimate the error of the numerical solutions with respect to the strong solution by both the discrete and continuous versions of the relative entropy functional, and show that the continuous approach is better in two folds:

- It provides a better convergence rate;
- It works for all $\gamma > 1$ and thus removes the limitation of $\gamma > \frac{3}{2}$ which is needed in the discrete approach.

Homogenization of the evolutionary compressible Navier–Stokes–Fourier system in domains with tiny holes

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We study the homogenization of the evolutionary compressible Navier–Stokes–Fourier system in a bounded three-dimensional domain, which is perforated with a large number of very tiny holes. We can show that under suitable assumptions on the smallness and the distribution of the holes, the limit system in the unperforated domain remains the same.

One of the main novelties here is in the treatment of the entropy inequality, which also improves the related result for the steady case treated by Y. Lu, and M. Pokorný (2021).

Fast multiplication, determinants, inverses, and eigenvectors of arrowhead and diagonal-plus-rank-k matrices over non-commutative algebras

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All matrices condisered are in $\mathbb{F}^{n\times n}$ where $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{R}^{k\times k}\}$, where $\mathbb{H}$ is the field of quaternions. Arrowhead matrix is a matrix of the form

$$A = \begin{bmatrix} D & u \\ v^* & \alpha \end{bmatrix}$$

where $\operatorname{diag}(D), u, v \in \mathbb{F}^{n-1}$, $\alpha \in \mathbb{F}$, or any symmetric permutation of such matrix. Diagonal-plus-rank-one (DPR1) matrix is a matrix of the form

$$A = \Delta + x\rho y^*$$

where $\operatorname{diag}(\Delta), x, y \in \mathbb{F}^n$, $\rho \in \mathbb{F}$. Both types of matrices allow $O(n)$ multiplication with the vector for every $\mathbb{F}$.

For both types of matrices, determinants can be computed in $O(n)$ operations [2,3]. The formulas depend on whether all elements D or Δ are non-zero or not. For quaternionic matrices, the same formulas are used to compute the Study determinant [1].

The inverses of both types of matrices are structured: the inverse of an arrowhead matrix is a DPR1 matrix if D is non-singular D, and a permuted arrowhead matrix otherwise. The inverse of a DPR1 matrix is a DPR1 matrix if Δ is non-singular, and

an arrowhead matrix otherwise. In all cases the same formulas are elegantly used for any $\mathbb{F}$, making use of the multiple dispatch feature of the programming language Julia.

Eigenvector matrices of arrowhead and DPR1 matrices have a Cauchy-like structure and can be represented via generators. The product of two linked Cauchy-like matrices is again a Cauchy-like matrix, and its generators are computed in $O(n)$ operations [4].

The above results are applied to fast $O(n^2)$ eigenvalue computations of arrowhead and DPR1 matrices using modified Rayleigh quotient iterations and Cauchy-like structure of eigenvector matrices [4,5]. The results are also applied to fast updating of the singular value decomposition [2].

References:

[1] H. Aslaksen. Quaternionic determinants. *The Mathematical Intelligencer*, 18: 57–65, 1996

[2] N. Jakovčević Stor, I. Slapničar and J. L. Barlow. Accurate eigenvalue decomposition of real symmetric arrowhead matrices and applications. *Linear Algebra and its Applications*, 464: 62-89, 2015

[3] N. Jakovčević Stor, I. Slapničar and J. L. Barlow. Forward stable eigenvalue decomposition of rank-one modifications of diagonal matrices. *Linear Algebra and its Applications*, 487: 301-315, 2015

[4] N. Jakovčević Stor, I. Slapničar and Z. Tomljanović. Fast computation of optimal damping parameters for linear vibrational systems. *Mathematics* 10(5): 790, 2022.

[5] H. Schwetlick and U. Kandler. Block Newton Method and Block Rayleigh Quotient Iteration for computing invariant subspaces of general complex matrices. *Linear Algebra and its Applications*, 526: 60-94, 2017

Low-rank tensor approximations for solving multi-marginal optimal transport problems

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By adding entropic regularization, multi-marginal optimal transport problems can be transformed into tensor scaling problems, which can be solved numerically using the multi-marginal Sinkhorn algorithm. The main computational bottleneck of this algorithm is the repeated evaluation of marginals. In [Haasler et al., *IEEE Trans. Inf. Theory*, 67 (2021)], it has been suggested that this evaluation can be accelerated when the application features an underlying graphical model. In this work, we accelerate the computation further by combining the tensor network dual of the graphical model with additional low-rank approximations. For the color transfer of images, these added low rank approximations save more than 96% of the computation time.

Modeling interaction of bodies of different dimensions

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In cases when a thin body is in contact to another body the thin body is often neglected. Rigorous asymptotic analysis reveals that this is not always true and that several possible models are possible, depending on the ratio of the material properties and the small thickness. Furthermore, it can be possible to build a model replacing the thin continua by a lower-dimensional model that approximates the limit models in all regimes.

Joint work with M. Ljulj, E. Marušić–Paloka and I. Pažanin.

Numerically Efficient H_∞ Analysis of Cooperative Multi-Agent Systems

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We propose a numerically efficient approach for computing the maximal (or minimal) impact a subset of agents has on the cooperative system. For example, if one is able to disturb/bolster several agents so as to maximally disturb/bolster the entire team, which agents to choose and what kind of inputs to apply? We quantify the agents-to-team impacts in terms of H_∞ norm whereas output synchronization is taken as the underlying cooperative control scheme. Moreover, we analyse in details impact that one agent has on the cooperative system it belongs to. Sufficient conditions on agents' parameters, synchronization gains and topology are provided such that the associated H_∞ norm attains its maximum for constant agents' disturbances. Linear second-order agent dynamics and weighted undirected topologies are considered. Our analyses also provide directions towards improving graph design and tuning/selecting cooperative control mechanisms. Lastly, numerical examples, some of which include forty thousand agents, are provided.

Global existence of weak solutions in nonlinear 3D thermoelasticity

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Here, I will talk about a nonlinear thermoelasticity hyperbolic-parabolic system describing the balance of momentum and energy of a heat-conducting elastic body in 3D. An equivalent system is introduced in which the energy balance is replaced with the entropy balance. For this system, a concept of weak solution is introduced which satisfies entropy inequality instead of balance and has a positive temperature almost everywhere. In our result, the global existence, consistency and weak-strong uniqueness are shown for two models – one having linear and other one having nonlinear heat energy and heat flux. This is the first result concerning global existence for large initial data in nonlinear thermoelasticity where the model is in full accordance with the laws of thermodynamics.

A free boundary inviscid model of flow-structure interaction

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We obtain the local existence and uniqueness for a system describing interaction of an incompressible inviscid fluid, modeled by the Euler equations, and an elastic plate, represented by the fourth-order hyperbolic PDE. We provide a priori estimates for the existence with the optimal regularity H^r, for $r > 2.5$, on the fluid initial data and construct a unique solution of the system for initial data $u_0 \in H^r$ for $r \geq 3$. An important feature of the existence theorem is that the Taylor-Rayleigh instability does not occur.

Numerical challenges connected with simulation of human phonation

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In this talk a two-dimensional mathematical model of human phonation is presented.

Voice production is a multi-physical complex process, which consists of three different physical fields – the deformation of the vocal folds (elastic body), the complex fluid flow and the acoustics together with mutual couplings. The flow-induced vibrations of (elastic) vocal folds generate a basic sound source, which is modified by the acoustic resonances of vocal tract cavities. The typical regime of vocal folds vibrations during a voice production is of a flutter type of instability. During the phonation a higher flow rate causes vibrations with increasing amplitudes, which leads to collisions of the vocal folds and to the closure of the glottis.

Consequently, the mathematical modelling of phonation process is challenging task, where besides accurate simulations of the flow field, the structural deformation and acoustics, the contact phenomena needs to be addressed properly in the model. This is rather difficult as it needs to be included not only to the structural model as impact forces but also to the fluid model, where it influences the geometrical domain deformation with possible topological changes as well as the artificial boundary conditions.

Finally, we addressed here the problem of fluid-structure interaction as the produced sound can be modelled by an acoustic analogy approach, effectively decoupling acoustics from the fluid-structure-acoustic interaction. Further, the air flow is described by the incompressible Navier-Stokes equations due to small velocities in glottal region and a simplified model of the elastic structure is considered. The main attention is paid to the problem of glottis closure. This is realized by a modification of the computational domain, use of an artificial porous media subdomain and a

suitable modification of the inlet boundary condition. The described mathematical model is discretized with the aid of the stabilized finite element method and numerical results are shown.

Control cost estimates for the heat equation on the sphere

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We show that the restriction of a polynomial to a sphere satisfies a Logvinenko-Sereda-Kovrijkine type inequality. This implies a spectral inequality for the Laplace-Beltrami operator, which, in turn, yields observability and null-controllability with explicit estimates on the control costs for the spherical heat equation that are sharp in the large and in the small time regime. This is joint work with Alexander Dicke.

Deep learning of the elliptic curves rank

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Let E be an elliptic curve over $\mathbb{Q}$ with discriminant Δ and conductor N. A celebrated Mordell's Theorem states that the group of rational points $E(\mathbb{Q})$ is a finitely generated abelian group isomorphic to $E(\mathbb{Q})_{tors} \times \mathbb{Z}^r$, where $E(\mathbb{Q})_{tors}$ is the torsion subgroup of $E(\mathbb{Q})$ and r is the rank of E. It is not known what values can rank attain and there is no agreement whether the rank is unbounded or not. Unfortunately, it is very computationally expensive to determine the rank of an elliptic curve. The curves of higher rank are hard to find and the current rank record is 28. One way around this is to use rank heuristics inspired by the Birch and Swinnerton-Dyer (BSD) conjecture to filter out probable candidates for high rank elliptic curves.

For each prime of good reduction $p \nmid \Delta$, we define $a_p = p + 1 - \#E(\mathbb{F}_p)$. For $p|N$, we set $a_p = 0, -1$, or 1 if, respectively, E has additive, split multiplicative or non-split multiplicative reduction at p. The L-function attached to E is then defined as an Euler product

$$L_E(s) = \prod_{p|\Delta} \left(1 - \frac{a_p}{p^s}\right)^{-1} \prod_{p\nmid\Delta} \left(1 - \frac{a_p}{p^s} + \frac{p}{p^{2s}}\right)^{-1},$$

which converges absolutely for $\Re(s) > 3/2$ and extends to an entire function by the Modularity theorem. BSD conjecture states that the order of vanishing of $L_E(s)$ at $s = 1$ (the quantity known as the analytic rank) is equal to the rank of E.

Mestre, Nagao, and later some others, motivated by BSD conjecture, considered certain sums involving the sequence of a_p-s, which heuristically should be able to detect curves of high analytic rank.

In this work, we develop a deep learning model for rank classification based on convolutional neural network architecture. Our model take as an input the conductor of an elliptic curve together with the sequence of a_p-s in a fixed range. Basically, our model can be considered as a highly parameterized heuristic for rank detection, where we optimize the parameters in regard to the quality of rank classification. We compare its performance to that of Mestre-Nagao sums.

Our approach definitely outperformed Mestre-Nagao sums.

Generalised solutions to linear and non-linear Schrödinger-type equations

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We consider a semi-linear Schrödinger equation of Hartree type in three spatial dimensions and its various approximations of singular, point-like perturbations. The corresponding nets of approximate solutions represent generalised solutions for the singular-perturbed Schrödinger equation. The behaviour of such nets is investigated.

We also study a generalised solution in the Colombeau algebra and for such a solution compatibility with the classical Hartree equation is established, in the sense of the Colombeau generalised solution theory.

This is joint work with Nevena Dugandžija and Alessandro Michelangeli.

An existence result for two-phase two-component flow in porous medium by the concept of the global pressure

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We derive an existence result for the initial-boundary value problem describing two-phase two-component flow in porous media. Firstly, we introduce the global pressure in the case of compositional flow following [2], based on the global pressure introduced in [1]. By rewriting equations in the terms of the global pressure and gas-pseudo pressure as primary unknowns, we prove the existence of weak solutions in a more tractable way than it is done in [3]. The usage of global pressure formulation allows us to avoid certain regularizations used in [3].

References:

[1] B. Amaziane, M. Jurak, *A new formulation of immiscible compressible two-phase flow in porous media*, C. R. Mecanique 336, 600-605 (2008).

[2] B. Amaziane, M. Jurak, A. Žgaljić Keko, *Modeling compositional compressible two-phase flow in porous media by the concept of the global pressure*, Comput Geosci 18, 297-309 (2018).

[3] M. Jurak, I. Radišić, A. Žgaljić Keko: *Two-phase Two-component Flow in Porous Media in Low Solubility Regime*, SIAM J. Math. Anal., 51(3), 2019-2052 (2019).

Local extrema in optimal design problems

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We consider conductivity optimal design problems for two isotropic phases, possibly with several state equations. Our aim is to find a distribution of materials such that the energy functional is optimized. For the minimization problem the perimeter of the interface between phases is added, as the regularization term.

We analyze optimality conditions obtained by shape derivatives. For problems on a ball, the first-order optimality condition easily leads to few critical shapes. By using classical Fourier analysis techniques we are able to express and analyze the second-order optimality conditions, and identify some local extrema of the functional.

Quasilinear and Time-Dependent Implicit, Degenerate Equations in Poro-elasticity

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Poro-elastic systems—bulk 3D elasticity as well as 2D plate models—have received recent attention for their application to biological systems. Most biological tissues are both porous and elastic, thus there is applied interest in studying Biot systems, especially those for which the permeability depends on the local fluid content. This scenario translates to a quasilinear, elliptic-parabolic system, with solution-dependent permeability. The resulting PDE system can be rewritten as a single *implicit, degenerate evolution equation*, for which a general theory has been developed over the past 30 years. In this talk, we describe recent well-posedness of weak solutions for the quasilinear 3D Biot system, as well as for its 2D counterpart. It is of particular note that even the linear Biot problem with time-dependent permeability—upon which fixed-point and discretization approaches are based—exhibits stark issues for weak solutions. These have only been resolved quite recently. We begin with the general theory of weak solutions in this context, and present recent results for such poro-elastic systems.

A High-Order-Integration Scheme for Global Polynomial Representations of Closed Surfaces

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We provide an extension of classic *finite element–(curved) triangle integration methods*, approximating a broad class of surface integrals with higher efficiency and accuracy. The approach rests on recently established results [1,2] that enable to implicitly represent a broad class of closed surfaces $S = Q_S^{-1}(0)$ by a *global polynomial level set* (zero contour). While classic surface integration [3] is based on approximating the surface

$$S \approx \mathcal{M} = \bigcup_{T \in \Omega} T$$

by a piecewise smooth manifold given as the union of individual triangles, the accuracy of the integration is limited to *low order approximations*.

Here, we present a *high-order-approximation* being applicable for non-standard surfaces, thereby crossing the limitations of established *curved triangle techniques* [4,5]. The approximation is given by incorporating novel regression techniques [2] in order to construct a piecewise smooth *Lagrange–polynomial approximation* $\bar{\Phi} \approx \Phi$ of

a piecewise diffeomorphism $\Phi : \mathcal{M} \longrightarrow S$. Given $\bar{\Phi}$, integration is realized straight forward by applying transformed Gauss quadrature rules for each of the triangles

$$\int_{\mathcal{S}} f dS \approx \int_{\mathcal{M}} \Phi^* f dM \approx \sum_{T \in \Omega} \int_T \bar{\Phi}^* f dT = \sum_{T \in \Omega} \sum_{p \in T} f(p) \omega_p \,. \tag{3}$$

The close approximation $\bar{\Phi} \approx \Phi$ we, hereby, provide enables accurate computations of the weights ω_p accordingly to the linear transformation induced by the Jacobian $\mathrm{Jac}(\Phi)$ of Φ.

While we estimate the limitations of our approach in terms of reachable surface complexity theoretically, they are intuitively given by the geometry of the prominent *Stanford bunny dataset*. This suggests our approach to deeply impact the methodology of numerical computations on complex geometries, such as ODE & PDE problems modeling dynamic surface deformations.

References:

[1] Hecht, Michael and Gonciarz, Krzysztof and Michelfeit, Jannik and Sivkin, Vladimir and Sbalzarini, Ivo F (2020), *Multivariate Interpolation in Unisolvent Nodes–Lifting the Curse of Dimensionality.*

[2] Hernandez Acosta, Uwe and Krishnan Thekke Veettil, Sachin and Wicaksono, Damar and Hecht, Michael(2021), *Multivariate Interpolation in Python*, https://github.com/casus/minterpy/

[3] Sander, Oliver (2020), *DUNE — The Distributed and Unified Numerics Environment*

[4] Simon Praetorius and Florian Stenger (2020), *Dune-CurvedGrid – A Dune module for surface parametrization*

[5] Dziuk, Gerhard and Elliott, Charles M. (2013) *Finite element methods for surface PDEs*

Numerical solution of conservation laws using second and third order accurate semi-implicit WENO schemes

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In this talk, we deal with the numerical solution of the linear advection equation with variable velocity using a novel semi-implicit time discretization. The method can be viewed as an alternative and high-order extension to the second order accurate Crank-Nicolson type of time discretization. The method can be easily introduced by using a partial Cauchy-Kowalevski (or Lax-Wendroff) procedure where the time derivatives in Taylor series are replaced by mixed derivatives of the solution exploiting the partial differential equations. Moreover, we use the inverse Lax-Wendroff procedure to approximate the boundary conditions.

We present the details of the third order accurate schemes in the case of smooth solutions. To suppress unphysical oscillations in the case of nonsmooth solutions, we derive the WENO form of the scheme. All presented numerical schemes are based on finite volume method (FVM) and are applied to some representative one-dimensional nonlinear conservation laws.

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Numerical solution of nonlinear hyperbolic systems of equations using high-resolution semi-implicit schemes

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This talk deals with the numerical solution of nonlinear conservation laws using a novel class of semi-implicit schemes as published for linear advection equations (P. Frolkovič and K. Mikula, "Semi-implicit second-order schemes for the numerical solution of level set advection equation on Cartesian grids", Applied Mathematics and Computation, vol. 329, pp. 129-142, 2018). We present an extension of the method for nonlinear hyperbolic systems like shallow water equations and Euler equations. We use some standard and newly developed TVD limiters to suppress unphysical oscillations for nonsmooth solutions. The resulting nonlinear scheme is unconditionally stable, and it requires only an application of a local nonlinear algebraic solver due to the nonlinearity in the flux function and the dependence of the limiter on the numerical solution. The scheme can be viewed as a second-order

accurate extension of the first order accurate numerical scheme. We use the so-called method of fractional steps, where the flux function is split into positive part, where the eigenvalues have only the positive values and into the negative part with only negative values of eigenvalues.

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